2.5 ~ Double Angle Formulas

and Half-Angle Formulas

- Develop and use the double and half-angle formulas.
- Evaluate trigonometric functions using these formulas.
- Verify identities and solve more trigonometric equations.

$\sin(2u) = \sin(u + u)$

$\cos(2u) = \cos(u + u)$

$\tan(2u) = \tan(u + u)$

Why do we need these? Do they give us functions of new angles?

Example 1: Solve an equation with *2x.*

$$\sin(2x) + \cos x = 0$$

POWER REDUCING FORMULAS
HALF-ANGLE FORMULAS

What about these?

$\sin(\frac{u}{2})$ $\cos(\frac{u}{2})$ $\tan(\frac{u}{2})$

Example 2: Use the formulas to compute the exact value of each of these.

a) $\sin 105°$

b) $\tan \frac{3\pi}{8}$

Example 3: Evaluate these expressions involving double or half angles.

If $\sin\theta = \frac{5}{13}$, find $\sin(2\theta)$, $\cos\left(\frac{\theta}{2}\right)$ and $\tan(2\theta)$.

Example 4:

Here is a problem you can work in two ways with very different results. Are they the same?

Find $\cos\left(\frac{7\pi}{12}\right)$.

a) using a half-angle formula:

b) using a sum/difference formula:

Other formulas to be aware of:

Product-to-Sum Identities

$$\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$

$$\sin(a)\sin(b) = \frac{1}{2}\left(\cos(a-b) - \cos(a+b)\right)$$

$$\sin(a)\cos(b) = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$$

$$\cos(a)\sin(b) = \frac{1}{2}\left(\sin(a+b) - \sin(a-b)\right)$$

Sum-to-Product Identities

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$