Inverse Trigonometric Functions

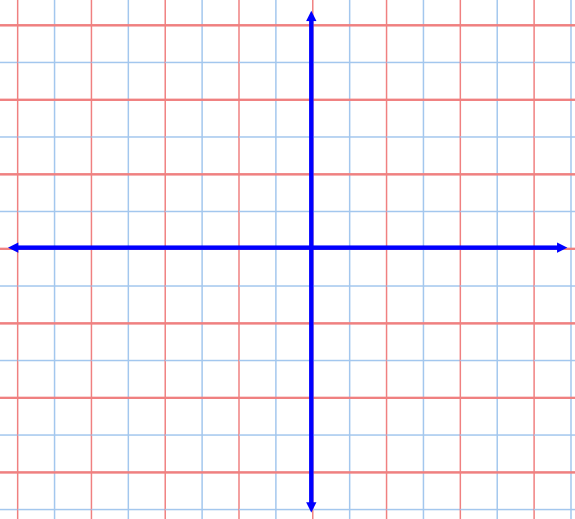
You will learn to:

Evaluate and graph the inverse sine function.
Evaluate and graph the other inverse trigonometric functions.

The inverse of a function $f(x)$ is written $f^{-1}(x)$, pronounced f inverse of x.

The -1 is NOT an exponent.
The original function must be 1-to-1.
The inverse is a reflection through the line y = x
An (a,b) pair on the function becomes a (b,a) pair on the inverse.
The domain of $f(x)$ is the range of $f^{-1}(x)$ and visa versa.

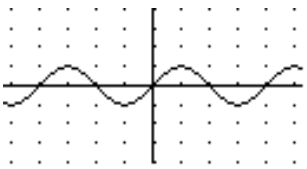
Example: Inverse of $y = x^2$



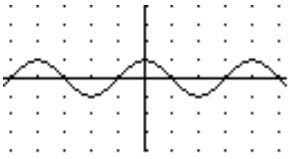
y = sin x

y = cos x

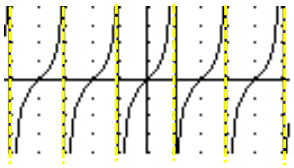
y = sin x



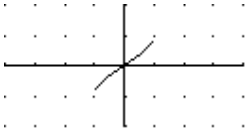
y = cos x



y = tan x

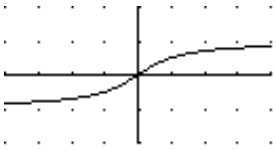


$y = \sin^{-1} x$



$y = \cos^{-1} x$

$y = \tan^{-1} x$



The important thing to remember is the answer to a question about an inverse function is unique and must come from a certain range.

Arcsin x must have an answer in the interval $[-\pi/2, \pi/2]$.

As will the arccsc x, arctan x and arccot x functions.

Arccos x must have an answer in the interval $[0, \pi]$.

As will the arcsec x function.

Try these:

$\sin^{-1}(\sqrt{3}/2)$

$\sin^{-1} -\frac{\sqrt{2}}{2}$

$\cos^{-1} 0$

$\sec^{-1}(-2/\sqrt{3})$

$\tan^{-1}(-1/\sqrt{3})$

$\sec^{-1} -1$

$\cos^{-1}(\sqrt{3}/2)$

$\tan^{-1} -1$

$\sin^{-1}(-1/2)$

$\sec^{-1} -\sqrt{2}$

$\tan^{-1} -\sqrt{3}$

$\cos^{-1} -\frac{\sqrt{2}}{2}$

Some more complex problem involving arcsin, arccos and arctan:
Hint: Draw a right triangle!

a) cos (arctan (2/3))

b) tan ($\sin^{-1}(3/4)$)

c) sec (arcsin x)

d) csc ($\tan^{-1}(3x/2)$)

And a few more:

a) sec (arctan(-3/4))

b) $\cot(\sin^{-1}(-0.2))$

c) A plane flies at an altitude of 6 miles toward a point directly over an observer. Write the angle ø as a function of x, the horizontal distance from the observer to a point on the ground directly below the airplane.