Math 1050 ~ College Algebra
9 Real Zeros of Polynomials

We are now ready to determine the rational roots of a polynomial.

Rational Zeros Theorem

If $f(x)$ is a polynomial that has integer coefficients, every rational zero of $f(x)$ has the form $\frac{p}{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient.

EX 1

Use the Rational Zeros Theorem to determine the possible roots of these functions.

1a)

$f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$

1b)

$g(x) = 3x^3 + 3x^2 - 11x - 10$

This rule may further help you in eliminating some of the options when determining the roots of a polynomial.

Descartes Rule of Signs

Given a polynomial function with real coefficients and a constant term not zero:

- The number of positive real roots is equal to the number of variations in signs of $f(x)$ or less than that by an even number.
- The number of negative real roots is equal to the number of variations in signs of $f(-x)$ or less than that by an even number.

EX 2

Determine how many positive and negative roots these functions are likely to have.

2a)

$f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$

2b)

$g(x) = 3x^3 + 3x^2 - 11x - 10$

EX 3

Find all zeros for these functions.

3a)

$f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$

3b)

$g(x) = 3x^3 + 3x^2 - 11x - 10$

Multiplicity of Roots

A factor $(x - a)^k, k > 1$, yields repeated zero $x = a$ of multiplicity k.

EX 4

Determine the roots and state the multiplicity of each. Write in factored form. $f(x) = x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8$.