Math 1050 ~ College Algebra

7 Graphs of Polynomials

A Polynomial Function and Vocabulary

$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_l x + a_0.$

Degree

Leading Term

Leading coefficient

Constant

EX 1

Determine which of these are polynomial functions and identify the degree, the leading term, the leading coefficient and the constant of those that are.

1a)

$f(x) = \sqrt{5}x^2 - 4x^3$

1b)

$f(x) = \sqrt[3]{x+2} + 1$

1c)

$f(x) = -3(x-2)^2 + 4x^6$

1d)

$f(x) = \frac{x-3}{x+2}$

1e)

$f(x) = \frac{6x^5+3x^2-1}{3}$

1f)

$f(x) = \pi$

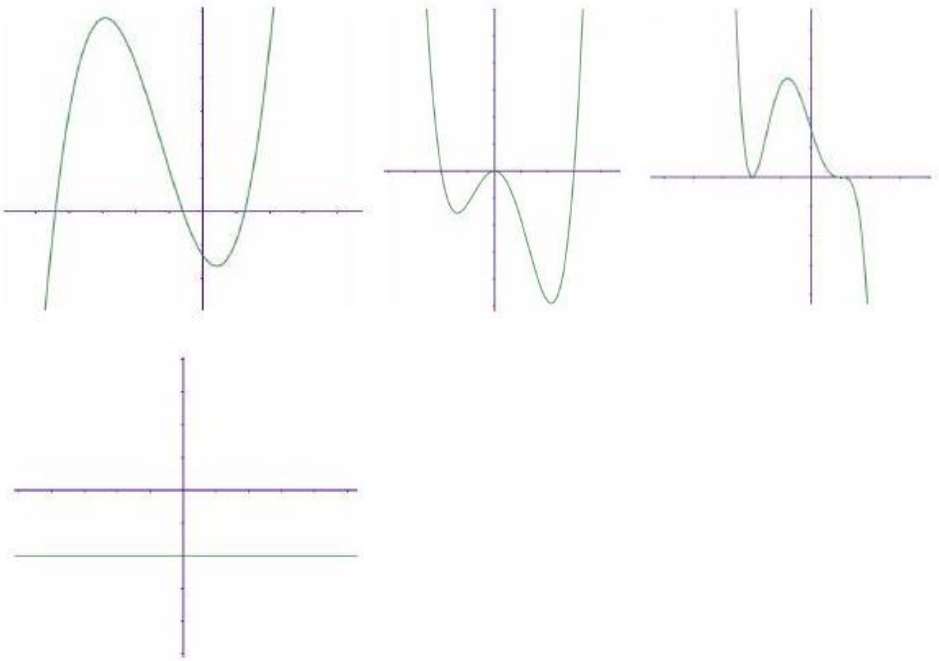
Polynomial functions have the characteristic of being continuous and smooth.
The leading coefficient and the degree can tell us a lot about the graph of a polynomial, including its end behavior.

n is odd, $a > 0$ n is even, $a < 0$

n is even, $a > 0$ n is odd, $a < 0$

EX 2

For each graph, guess at a likely degree, circle the x and y-intercepts, and identify the sign (+ or -) of the leading coefficient.



To graph a polynomial, it helps to determine the roots and the y-intercept.

Real Zeros of Polynomial Functions

Equivalent Statements: for $a \in \Re, f(x)$ a polynomial

- $x = a$ is a zero of $f(x)$.
- $x = a$ is a solution of $f(x) = 0$.
- $(x - a)$ is a factor of $f(x)$.
- $(a, 0)$ is an x-intercept of the graph of $f(x)$.

Repeated Zeros

A factor $(x - a)^k$ for $k > 1$ yields a repeated zero, $x = a$ of multiplicity k.

- If k is odd, the graph crosses the x-axis at $x = a$.
- If k is even, the graph touches the x-axis at $x = a$.

Intermediate Value Theorem

Let $a, b \in \Re$ and $a < b$. If $f(x)$ is a polynomial and $f(x) \neq f(b)$, then over the interval $[a, b], f$ takes on every value between $f(a)$ and $f(b)$.

EX 3

For each function, describe the end-behavior, find all real zeros, including multiplicity, and the number of turning points on the graph.

3a)

$f(x) = (x + 2)^2(x - 1)^3$

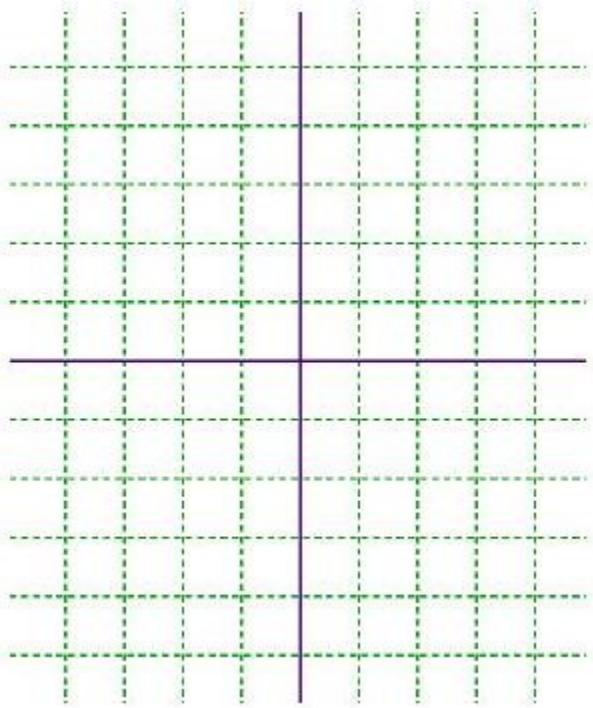
3b)

$f(x) = -x(x+7)(x-3)^2$

EX 4

Sketch the graph of f(x) by looking at the leading coefficients, finding the zeros, and perhaps plotting more points.

$f(x) = -48x^2 + 3x^4$



An Application Problem

EX 5

The profit (in millions of dollars) for a sport cap manufacturer can be modeled by $P(x) = -x^3 + 4x^2 + x$, where x is the number of caps they produce (in millions). They currently produce 4 million caps, making a profit of $4,000,000. What smaller number of caps could they make and still make the same profit?