

# Math 1050 ~ College Algebra

## 6 Quadratic Functions

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

### Learning Objectives

- Graph a quadratic function through transformations of  $f(x) = x^2$ .
- Change a quadratic function from general to standard form.
- Find the vertex and axis of symmetry of a quadratic function.
- Find the intercepts of a quadratic function.
- Graph a quadratic function using vertex, axis of symmetry and intercepts.
- Solve applications that require finding the maximum or minimum value of a quadratic function.

### Quadratic Functions

A polynomial function:  $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$ .

A quadratic function is a type of polynomial function where the degree = 2.

$$f(x) = ax^2 + bx + c \quad a, b, c \in \mathcal{R}, a \neq 0$$

general form

$$f(x) = ax^2 + bx + c$$

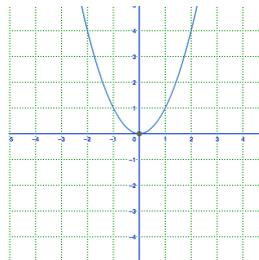
standard form

$$f(x) = a(x - h)^2 + k$$

axis of symmetry

vertex

concavity

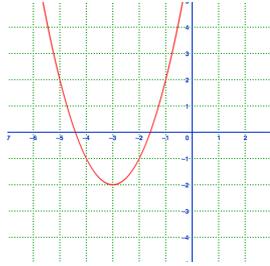


Ex 1: Determine the vertex, axis of symmetry and concavity of each of these.

a)  $f(x) = 3x^2 + 6x - 4$

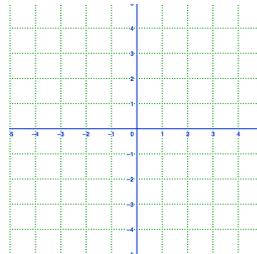
b)  $f(x) = -2(x+3)^2 - 4$

Ex 2: Write the equation of this quadratic function in standard form, then use algebra to write it in general form.



Ex 3: Put this equation in standard form and sketch a graph of it.

$$y = -2x^2 + 4x + 2$$



### **Finding Roots of Quadratic Functions**

To find the roots, solve for  $f(x) = 0$ .

If the expression on the left factors, set each factor equal to 0 and solve for  $x$ .

If you prefer not to factor, or it does not factor, you can always use the Quadratic Formula.

Quadratic Formula      $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex 4: Determine the roots of each of these.

a)  $f(x) = 3x^2 + 5x - 4$      b)  $f(x) = 9x^2 - 6x + 1$      c)  $f(x) = 4x^2 - 6x - 3$

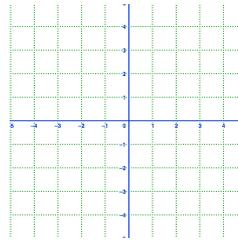
In the quadratic formula, the expression inside the radical is called the discriminant. It determines whether there is one real root, two real roots or no real roots.

Ex 5: Find the discriminant of the equations in example 4.

a)  $f(x) = 3x^2 + 5x - 4$       b)  $f(x) = 9x^2 - 6x + 1$       c)  $f(x) = 4x^2 - 6x - 3$

Ex 6: For this function, find the vertex, axis of symmetry,  $x$  and  $y$ -intercepts and sketch it.

$$f(x) = -\frac{1}{2}(x^2 - 10x + 21)$$



### An Application Problem

Ex 7: The height of an object shot straight up in the air from a height of 128 feet at an initial velocity of 32 ft/sec is modeled by  $h(t) = -16t^2 + 32t + 128$ , where  $t$  = time.

Determine the maximum height the object reaches and the time it will hit the ground.