



# Math 1050 ~ College Algebra

## 4 Combinations of Functions

### Learning Objectives

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

- Find and simplify functions involving arithmetic expressions.
- Combine functions through addition, subtraction, multiplication and division.
- Determine the domain of a function resulting from an arithmetic operation.
- Find the difference quotient of a function.
- Create a new function through composition of functions.
- Find the domain of a composite function.
- Find values of composite functions.
- Decompose a composite function into its component functions.

Ex 1: Evaluate this function at the given expressions, simplifying your answer.

$$f(x) = x^2 - 4x + 3$$

- a)  $f(-3)$                       b)  $f(x-2)$                       c)  $f(x^2)$                       d)  $f(x^2+1)$

It is also possible to perform arithmetic operations on functions.

Sum

Difference

Product

Quotient

Composition

Ex 2: For  $f(x) = \sqrt{x-1}$ , and  $g(x) = \frac{x}{x^2-4}$ , simplify the resulting function and determine the domain.

a)  $(f+g)(x)$

b)  $(f-g)(5)$

c)  $\left(\frac{f}{g}\right)(x)$

Ex 3: For the two functions above, find

a)  $f(g(x))$  (include domain)

b)  $g(f(x))$  (include domain)

Ex 4: For  $f(x) = 3x + 5$ , find  $(f \circ f)(x)$  and its domain.

In calculus, one frequently is required to find a difference quotient,  $\frac{f(x+h)-f(x)}{h}$ .

Ex 5: Find the difference quotient for each of these.

a)  $f(x) = 3x + 5$

b)  $f(x) = x^2 - 3x + 1$

## Decomposing Functions

Ex 6: Find two functions  $f$  and  $g$  such that  $f(g(x)) = h(x)$  where  $h(x) = \frac{3}{(5x+1)^2}$ .