

# Math 1050 ~ College Algebra

## 30 Binomial Expansion

### Binomial Expansion

The square and cube of a binomial are used so frequently, we may want to memorize them.

#### EX 1

$$(a + b)^2 =$$

$$(a + b)^3 =$$

Given these, it is possible to determine the square or cube of more complicated binomials.

#### EX 2

$$(x - 2y)^2 =$$

$$(3x + 2y)^3 =$$

### Pascal's Triangle

$$(a + b)^0 =$$

$$(a + b)^1 =$$

$$(a + b)^2 =$$

$$(a + b)^3 =$$

$$(a + b)^4 =$$

### EX 3

Using the pattern above, expand these.

**3a)**

$$(x - y)^5$$

**3b)**

$$(2x - 3y)^4$$

## The Binomial Theorem

To get to this theorem, we need to introduce a new notation.

Factorial

The factorial of a non-negative integer is defined as:

$$\begin{aligned} 0! &= 1 \\ n! &= 1(2)(3) \dots (n) \end{aligned}$$

### EX 4

Evaluate the factorials of the integers 1 through 6.

## EX 5

Evaluate

**5a)**

$$\frac{5!}{3!}$$

**5b)**

$$(8 - 5)!$$

**5c)**

$$(8 - 3)!$$

**5d)**

$$\frac{17!}{14!}$$

**5e)**

$$\frac{100!}{96!4!}$$

The Binomial Coefficient  $\binom{n}{k}$   $n, k$  are nonnegative integers with  $n \geq k$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

## EX 6

Evaluate these binomial coefficients.

**6a)**

a)  $\binom{5}{0}$

b)  $\binom{10}{6}$

c)  $\binom{8}{3}$

d)  $\binom{8}{5}$

e)  $\binom{8}{8}$

**EX 7**

Show that  $(a + b)^4 = \sum_{k=0}^4 \binom{4}{k} a^{4-k} b^k$

The Binomial Theorem  $(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$

**EX 8**

Use this theorem to expand  $(3x - 2)^5$ .

**EX 9**

Consider  $(2x - y)^8$ . Find the term that contains  $x^3$  and the term that contains  $y^4$ .