Math 1050 ~ College Algebra

29 Series

Summation Notation

$$\sum_{n=1}^{p} a_n = a_1 + a_2 + a_3 + \cdots + a_p$$

$$\sum_{n=j}^{p} a_n = a_j + a_{j+1} + a_{j+2} + \cdots + a_{p-1} + a_p$$

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

EX 1

Find the following sums.

1a)

$\sum_{n=2}^{6} (2n - 1)$

1b)

$\sum_{k=1}^{4} (-1)^k (2k)$

1c)

$\sum_{k=0}^{5} 2^k$

EX 2

Write the following sums using summation notation. Assume the terms in each result from an arithmetic or geometric sequence.

2a)

$9 - 6 + 4 - \frac{8}{3} + \frac{16}{9}$

2b)

$\frac{19}{2}+\frac{11}{2}+\frac{3}{2}-\frac{5}{2}+\cdots-\frac{29}{2}$

2c)

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$

Properties of Summation

$$\sum_{n=j}^{p}(a_n \pm b_n)=\sum_{n=j}^{p}a_n \pm \sum_{n=j}^{p}b_n$$

$$\sum_{n=j}^{p}a_n=\sum_{n=j}^{h}a_n+\sum_{n=h}^{p}a_n, \quad \text{for any integer } j \le h < p$$

$$\sum_{n=j}^{p}ca_n=c\sum_{n=j}^{p}a_n \quad c \text{ is a constant}$$

$$\sum_{n=j}^{p}a_n=\sum_{n=j+h}^{p+h}a_{n-h} \quad \text{for any integer } h\text{(if } p=\infty\text{, replace } p+h \text{ with } \infty)$$

EX 3

Use the properties above to state these in another way.

3a)

$\sum_{k=1}^{8} \frac{k^2}{3}$

3b)

$\sum_{k=1}^{10} \left(2k - \frac{1}{k^2}\right)$

3c)

$\sum_{j=2}^{5} (j+1) + \sum_{j=2}^{5} \frac{2}{j^2}$

Arithmetic Series

EX 4

Add the first hundred integers.

Sum of a Finite Arithmetic Sequence

$S_n = \sum_{j=1}^{n} a_j = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(2a_1 + (n-1)d), n \geq 2$ where $a_j = a_1 + (j-1)d$

EX 5

Find the n^{th} partial sum for each of these.

5a)

$\sum_{n=2}^{20} (2n-1)$

5b)

$\frac{19}{2} + \frac{11}{2} + \frac{3}{2} - \frac{5}{2} + \cdots, n = 10$

5c)

The sequence $\{a_n\}$ where $a_1 = 15, a_{10} = 312, n = 50$

Sum of a Finite Geometric Sequence

$$S_n = \sum_{j=1}^{n} a_j = a_1 \frac{(1-r^n)}{1-r} \text{ where } a_j = a_1(r^{j-1})$$

Sum of an Infinite Geometric Sequence

$S = \sum_{j=1}^{\infty} a_j = \frac{a_1}{1-r}, \quad -1 < r < 1$ where $a_j = a_1(r^{j-1})$

EX 6

Determine each sum.

6a)

$\sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$

6b)

$\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n$

6c)

$1.3\overline{8}$ Hint: $1.3\overline{8} = 1.3 + 0.08 + 0.008 + 0.0008 + \cdots$

6d)

$\sum_{k=0}^{5} 3^k$

Applications of Series

EX 7

You are trying to break a bad habit. Two relatives offer to help with a financial incentive, but you must choose only one. How much is each offer? Which would you take?

7a)

Your Great Auntie Mare offers to give you $1.00 on the first day of February and each day thereafter, she will give you one dollar more than she did the day before.

7b)

Your Uncle Ulysses offers to give you 1 cent on the first day of February and each day thereafter, he will give you double what he gave you the day before.