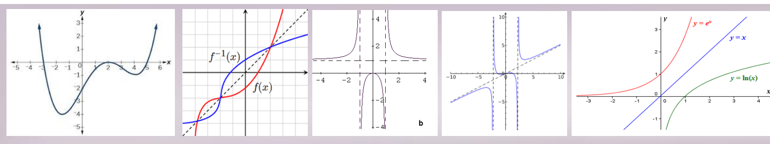

Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

29 Series

Learning Objectives

- Use summation notation.
- Find the sum of a finite arithmetic sequence.
- Solve applications of arithmetic series.
- Find the value of an infinite geometric series with a finite sum.
- Find the sum of a finite geometric sequence.
- Solve applications of geometric series.

Summation Notation

$$\sum_{n=1}^{p} a_n = a_1 + a_2 + a_3 + \cdots + a_p$$

$$\sum_{n=j}^{p} a_n = a_j + a_{j+1} + a_{j+2} + \cdots + a_{p-1} + a_p$$

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

Ex 1: Find the following sums.

a) $\sum_{n=2}^{6} (2n-1)$ b) $\sum_{k=1}^{4} (-1)^k (2k)$ c) $\sum_{k=0}^{5} 2^k$

Ex 2: Write the following sums using summation notation. Assume the terms in each result from an arithmetic or geometric sequence.

a) $9 - 6 + 4 - \frac{8}{3} + \frac{16}{9}$ b) $\frac{19}{2} + \frac{11}{2} + \frac{3}{2} - \frac{5}{2} + \cdots - \frac{29}{2}$ c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$

Properties of Summation

$$\sum_{n=j}^{p}\left(a_n \pm b_n\right) = \sum_{n=j}^{p} a_n \pm \sum_{n=j}^{p} b_n$$

$$\sum_{n=j}^{p} a_n = \sum_{n=j}^{h} a_n + \sum_{n=h}^{p} a_n, \quad \text{for any integer } j \le h < p$$

$$\sum_{n=j}^{p} c a_n = c\sum_{n=j}^{p} a_n \qquad c \text{ is a constant}$$

$$\sum_{n=j}^{p} a_n = \sum_{n=j+h}^{p+h} a_{n-h} \qquad \text{for any integer } h \text{ (if } p=\infty, \text{ replace } p+h \text{ with } \infty)$$

Ex 3: Use the properties above to state these in another way.

a) $\displaystyle\sum_{k=1}^{8}\frac{k^2}{3}$ b) $\displaystyle\sum_{k=1}^{10}\left(2k-\frac{1}{k^2}\right)$ c) $\displaystyle\sum_{j=2}^{5}(j+1)+\sum_{j=2}^{5}\frac{2}{j^2}$

Arithmetic Series

Ex 4: Add the first hundred integers.

Sum of a Finite Arithmetic Sequence

$$S_n = \sum_{j=1}^{n} a_j = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(2a_1 + (n-1)d), \quad n \ge 2 \qquad \text{where } a_j = a_1 + (j\text{-}1)d$$

Ex 5: Find the n^{th} partial sum for each of these.

a) $\sum_{n=2}^{20}(2n-1)$

b) $\frac{19}{2} + \frac{11}{2} + \frac{3}{2} - \frac{5}{2} + \cdots \quad , n = 10$

c) The sequence $\{a_n\}$ where $a_1 = 15$, $a_{10} = 312$, $n = 50$

Sum of a Finite Geometric Sequence

$$S_n = \sum_{j=1}^{n} a_j = a_1 \frac{(1-r^n)}{1-r} \qquad \text{where } a_j = a_1(r^{j\text{-}1})$$

Sum of an Infinite Geometric Sequence

$$S = \sum_{j=1}^{\infty} a_j = \frac{a_1}{1-r}, \quad -1 < r < 1 \qquad \text{where } a_j = a_1(r^{j\text{-}1})$$

Ex 6: Determine each sum.

a) $\sum_{n=0}^{\infty}\left(\frac{1}{10}\right)^n$

b) $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n$

c) $1.3\overline{8}$ Hint: $1.3\overline{8} = 1.3 + 0.08 + 0.008 + 0.0008 + \ldots$

d) $\sum_{k=0}^{5} 3^k$

Applications of Series

Ex 7: You are trying to break a bad habit. Two relatives offer to help with a financial incentive, but you must choose only one. How much is each offer? Which would you take?

a) Your Great Auntie Mare offers to give you $1.00 on the first day of February and each day thereafter, she will give you one dollar more than she did the day before.

b)Your Uncle Ulysses offers to give you 1 cent on the first day of February and each day thereafter, he will give you double what he gave you the day before.