Math 1050 ~ College Algebra

27 Partial Fractions

Partial Fraction Decomposition

Distinct Linear Factors

There are times, in future math classes, when you would like to break a rational expression into a sum of simpler fractions. We will begin with a proper fraction, where the degree of the numerator is less than the degree of the denominator. The first step is to factor the denominator and write it as a sum of n terms for an n^{th} degree denominator.

$$\frac{p(x)}{q(x)} = \frac{A}{a_1x + b_1} + \frac{B}{a_2x + b_2} + \frac{C}{a_3x + b_3} + \cdots$$

EX 1

Determine A and B for this proper fraction. $\frac{3x-1}{x(x-4)} = \frac{A}{x} + \frac{B}{(x-4)}$

If the fraction is improper, we must do long division first.

EX 2

Write the partial fraction decomposition for this expression. $\frac{x^2+1}{x^2-x}$

Repeated Linear Factors

$$\frac{p(x)}{q(x)} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3} + \cdots + \frac{N}{(ax+b)^n}$$

EX 3

Resolve into partial fractions $\frac{2x^2+7x+4}{(x+1)^3}$.

Unique Irreducible Quadratic Factors

$$\frac{p(x)}{q(x)} = \frac{Ax+B}{a_1x^2+b_1x+c_1} + \frac{Cx+D}{a_2x^2+b_2x+c_2} + \cdots$$

EX 4

Write the partial fraction decomposition of $\frac{-x^3+4x^2-2x+6}{x^2(x^2+2)}$.

Repeated Irreducible Quadratic Factors

$$\frac{p(x)}{q(x)} = \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \frac{A_3x + B_3}{(ax^2 + bx + c)^3} + \cdots + \frac{A_nx + B_n}{(ax^2 + bx + c)^n}$$

EX 4

Write the partial fraction decomposition of $\frac{x^2+x+2}{(x^2+2)^2}$.