

Math 1050 ~ College Algebra

25 System of Linear Equations

Inverse Matrix

If A and B are square matrices, $n \times n$, such that $AB = BA = I_n$, then B is the inverse matrix of A and can be denoted as A^{-1} .

EX 1

Show that B is A^{-1} . $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

Process for finding an inverse matrix.

1. Augment A with I .
2. Perform row operations until the left side looks like I .
3. The right side will be A^{-1} .

EX 2

Determine the inverse of each matrix, if it exists.

2a)

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

2b)

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

Let's derive a formula for the inverse of a 2×2 matrix.

We can write a system of linear equations as a matrix equation $AX = C$, where A is a matrix of coefficients, X is the matrix of variables and C is the matrix of constants.

EX 3

Write this system of equations as a matrix equation.

$$2x + y = 4$$

$$5x + 3y = 6$$

EX 4

Using this fact from Ex. 1, $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$ find the solution to Ex. 3.

EX 5

Refer back to example 2 to solve these systems of equations.

5a)

$$\begin{aligned} 2x + 3y &= 0 \\ x + 4y &= -5 \end{aligned}$$

5b)

$$\begin{aligned} x - y &= 2 \\ x - z &= 3 \\ 6x - 2y - 3z &= 15 \end{aligned}$$

EX 6

Solve this system using the techniques of this lesson.

$$\begin{aligned}2x - 3y &= 8 \\ -4x + 6y &= -5\end{aligned}$$