

# Math 1050 ~ College Algebra

## 24 Matrix Arithmetic

### Definition of a Matrix

A matrix is an array of numbers with  $m$  rows and  $n$  columns. The size of a matrix is described as  $m \times n$ . This is called the dimension of a matrix.

#### EX 1

Determine the dimension of each matrix.

$$A = \begin{bmatrix} 1 & 0 & -5 \\ -2 & 5 & 0 \\ 1 & 4 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 7 & 3 & 6 \\ 0 & 2 & -1 & 4 \\ 3 & 5 & 1 & 0 \end{bmatrix} \quad C = [-3 \quad 5 \quad 1 \quad 8] \quad D = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 0 \\ 2 & 5 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 3 & -1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The elements of a matrix are called entries. An  $m \times n$  matrix has  $m \cdot n$  entries. Two matrices are equal only if they are the same dimension and each corresponding element is equal.

#### EX 2

If  $\begin{bmatrix} x & 2 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 4 & y \end{bmatrix}$ , solve for  $x$  and  $y$ .

# Operations with Matrices

Matrix Addition/Subtraction  $\Rightarrow A \pm B$

If two matrices are the same dimension, then addition or subtraction may be accomplished by adding/subtracting the corresponding entries of the two matrices.

$$\begin{aligned} A &= \begin{bmatrix} 1 & 0 & -5 \\ -2 & 5 & 0 \\ 1 & 4 & 2 \end{bmatrix} & B &= \begin{bmatrix} 3 & 7 & 3 \\ 0 & 2 & -1 \\ 3 & 5 & 1 \\ & & 0 \end{bmatrix} & C &= [-3 \quad 5 \quad 1 \quad 8] & D &= \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} \\ F &= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 0 \\ 2 & 5 & 2 \end{bmatrix} & G &= \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 3 & -1 \end{bmatrix} & I &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ & & \mathbf{O} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

## EX 2

Determine which matrices from example 1 can be added or subtracted, then perform each operation on them (Exclude the Identity matrix.)

Scalar Multiplication  $\Rightarrow kA$

A scalar is a real number. Multiplication of a matrix by a scalar,  $k$  is accomplished by multiplying each entry by that scalar.

## EX 3

Perform these operations.

**3a)**

$$5C$$

**3b)**

$$-3G$$

**3c)**

$$2I - 3A$$

# Properties of Matrix Addition

- Commutative Property: For all  $m \times n$  matrices,  $A + B = B + A$
- Associative Property: For all  $m \times n$  matrices,  $(A + B) + C = A + (B + C)$
- Identity Property: For all  $m \times n$  matrices,  $A + \mathbf{0} = \mathbf{0} + A = A$
- Inverse Property: Every  $m \times n$  matrix  $A$  has a unique additive inverse, denoted  $-A$ , such that  $A + (-A) = (-A) + A = \mathbf{0}$

## EX 4

Write an additive identity matrix and an additive inverse matrix for each of these.

**4a)**

$$G = \begin{bmatrix} 1 & 0 \\ -2 & 4 \\ 3 & -1 \end{bmatrix}$$

**4b)**

$$C = [-3 \quad 5 \quad 1 \quad 8]$$

# Multiplying Matrices

To multiply an  $m \times p$  and  $p \times n$  matrix, note that the number of columns in the first matrix must be equal to the number of rows in the second matrix. The result will be an  $m \times n$  matrix.

Let's begin by demonstrating what it means to multiply a row of one matrix by a column of another.

For the matrices  $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 5 & -6 \\ 7 & 0 \end{bmatrix}$   $R_1$  will denote row 1 of  $A$   
and  $C_1$  will denote column 1 of  $B$ .

To multiply  $R_1$  by  $C_1$  means this:  $R_1 C_1 = [1 \ 2] \begin{bmatrix} 5 \\ 7 \end{bmatrix} = 1(5) + 2(7) = 19$

## EX 5

Determine each of these for  $A$  and  $B$  above.

**5a)**

$$R_1 C_2$$

**5b)**

$$R_2 C_1$$

**5c)**

$$R_2 C_2$$

The result is  $AB = \begin{bmatrix} R_1 C_1 & R_1 C_2 \\ R_2 C_1 & R_2 C_2 \end{bmatrix}$ .

## EX 6

**6a)**

Write the matrix  $AB$  for Ex 5.

**6b)**

Compute  $BA$  for the same matrices.

**Matrix Multiplication:** Suppose  $A$  is an  $m \times p$  matrix and  $B$  is a  $p \times n$  matrix. Let  $R_i$  denote the  $i^{\text{th}}$  row of  $A$  and  $C_j$  denote the  $j^{\text{th}}$  column of  $B$ . The product of  $A$  and  $B$ , denoted  $AB$ , is the matrix defined by

$$AB = \begin{bmatrix} R_1C_1 & R_1C_2 & \cdots & R_1C_n \\ R_2C_1 & R_2C_2 & \cdots & R_2C_n \\ \vdots & \vdots & \cdots & \vdots \\ R_mC_1 & R_mC_2 & \cdots & R_mC_n \end{bmatrix}$$

**EX 7**

Perform multiplication on these.

**7a)**

$$[-3 \quad 4 \quad 0] \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

**7b)**

$$\begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix}^2$$

**7c)**

$$\begin{bmatrix} -1 & 3 & 6 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 6 & -2 \\ 3 & 5 & 1 \\ -3 & 4 & 0 \end{bmatrix}$$

**Properties of Matrix Multiplication** Let  $A$ ,  $B$  and  $C$  such that all of the matrix products below are defined and let  $r$  be a real number.

- Associative Property of Matrix Multiplication:  $(AB)C = A(BC)$
- Associative Property with Scalar Multiplication:  $r(AB) = (rA)B = A(rB)$
- Identity Property: For a natural number  $k$ , the  $k \times k$  identity matrix, denoted  $I_k$ , is a square matrix containing 1's down the main diagonal and 0's elsewhere. For a  $m \times n$  matrix  $A$ ,  $I_m A = A I_n = A$ .
- Distributive Property of Matrix Multiplication over Matrix Addition:

$$A(B \pm C) = AB \pm AC \text{ and } (A \pm B)C = AC \pm BC$$