Math 1050 ~ College Algebra

17 Properties of Logarithms

Common and Natural Logarithms

Base 10 is commonly used in logarithms. Thus, when no base is indicated, it is assumed to be base 10.
$\log\ x = \log_{10}\ x$

EX 1

Evaluate these.

1a)

$\log\ 1{,}000{,}000$

1b)

$\log\ (10^{-3})$

1c)

$\log\ 0.01$

1d)

log (a trillion)

Another base is the irrational number, e, called the natural base. This is written using ln x.
$\log_e x = \ln x$

EX 2

Evaluate these.

2a)

$\ln e$

2b)

$\ln e^{-3}$

2c)

$\ln e^{8}$

2d)

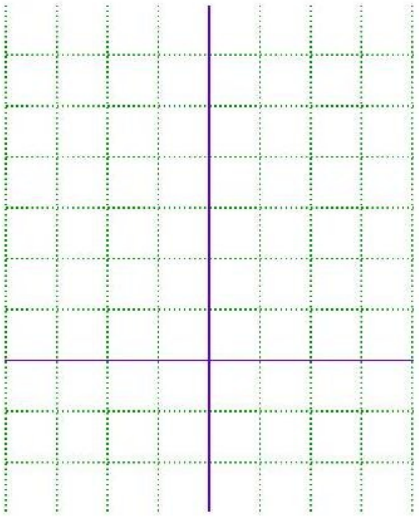
$\ln \left(\frac{1}{e^5}\right)$

Natural Exponential Base

$e \approx 2.718281828459 \ldots$

EX 4

Sketch a graph of $y = e^x$.



The exponential base is used in financial and scientific calculations which we will explore in a later chapter.

Logarithm Properties

Let b be a positive number, not equal to 1, and let x be a positive number.

$$\log_b\ 1 = 0$$
$$\log_b\ b = 1$$
$$\log_b\ b^x = x$$
$$\text{If } \log_b\ x = \log_b\ y, \text{ then } x = y$$

EX 5

Evaluate these.

5a)

$\ln\ 1$

5b)

$\log\ 100$

5c)

$\ln\ e^{\pi}$

5d)

$\log\ (10^{0.2})$

EX 6

Determine the value of x for each of these.

6a)

$\log\ x = \log\ (y + 5)$

6b)

$\ln\ x = \ln\ (\pi + 1)$

Properties of Logarithms

Change of Base Property

Let a and b be positive numbers, not equal to 1, and let x be a positive number.

$$\log_b\ x = \frac{\log_a\ x}{\log_a\ b}$$

EX 7

True or false? $\log_2\ 3 = \frac{\log\ 3}{\log\ 2}$

EX 8

Use your calculator to give an approximate value for these.

8a)

$\log_2\ 5$

8b)

$\log\ 50$

8c)

$\ln\ 8$

8d)

$\log_6\ 0.0002$

Inverse Properties

Let b be a positive number, not equal to 1.

$$b^{\log_b\ x} = x, \text{ for any positive number } x$$
$$\log_b\ b^x = x, \text{ for any real number } x$$

EX 9

Use the inverse properties to simplify.

9a)

$\ln\ e - 2$

9b)

$\log_5\ 1$

9c)

$6^{\log_6\ 20}$

9d)

$\log_3\ 3^{10}$

Exponent, Sum and Difference Properties of Logarithms

Let b be a positive number, not equal to 1, and let u and v be positive numbers.

$$\log_b\ (uv) = \log_b\ u + \log_b\ v$$
$$\log_b\ \left(\frac{u}{v}\right) = \log_b\ u - \log_b\ v$$
$$\log_b\ x^m = m\log_b\ x$$

EX 10

Use these properties to expand these expressions.

10a)

$\log \sqrt{x^2(x+2)}$

10b)

$\ln \left(\frac{x^2-1}{x^3}\right), x > 1$

EX 11

Use these properties to contract these expressions into a single term.

11a)

$3\log x + 4\log y - 5\log z$

11b)

$\frac{1}{2}[\ln (x+1) + 2\ln (x-1)] - 6\ln x$