

Math 1050 ~ College Algebra

17 Properties of Logarithms

Learning Objectives

- Use the definition of common and natural logarithms in solving equations and simplifying expressions.
- Use the change of base property to evaluate logarithms.
- Solve exponential equations using logarithmic properties.
- Combine and/or expand logarithmic expressions.
- Solve basic logarithmic equations using properties of logarithms and exponentials.

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

Common and Natural Logarithms

Base 10 is commonly used in logarithms. Thus, when no base is indicated, it is assumed to be base 10.

$$\log x = \log_{10} x$$

Ex 1: Evaluate these.

- a) $\log 1,000,000$ b) $\log (10^{-3})$ c) $\log 0.01$ d) \log (a trillion)

Another base is the irrational number, e , called the natural base. This is written using $\ln x$.

$$\log_e x = \ln x$$

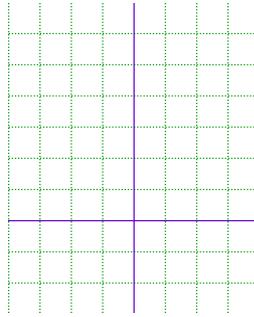
Ex 2: Evaluate these.

- a) $\ln e$ b) $\ln e^{-3}$ c) $\ln e^8$ d) $\ln\left(\frac{1}{e^5}\right)$

Natural Exponential Base

$$e \approx 2.718281828459\dots$$

Ex 4: Sketch a graph of $y = e^x$.



The exponential base is used in financial and scientific calculations which we will explore in a later chapter.

Logarithm Properties

Let b be a positive number, not equal to 1, and let x be a positive number.

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b b^x = x$$

$$\text{If } \log_b x = \log_b y, \text{ then } x = y$$

Ex 5: Evaluate these.

a) $\ln 1$

b) $\log 100$

c) $\ln e^\pi$

d) $\log (10^{0.2})$

Ex 6: Determine the value of x for each of these.

a) $\log x = \log (y + 5)$

b) $\ln x = \ln (\pi + 1)$

Properties of Logarithms

Change of Base Property

Let a and b be positive numbers, not equal to 1, and let x be a positive number.

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Ex 7: True or false? $\log_2 3 = \frac{\log 3}{\log 2}$

Ex 8: Use your calculator to give an approximate value for these.

- a) $\log_2 5$ b) $\log 50$ c) $\ln 8$ d) $\log_6 0.0002$

Inverse Properties

Let b be a positive number, not equal to 1.

$$b^{\log_b x} = x, \text{ for any positive number } x$$

$$\log_b b^x = x, \text{ for any real number } x$$

Ex 9: Use the inverse properties to simplify.

- a) $\ln e - 2$ b) $\log_5 1$ c) $6^{\log_6 20}$ d) $\log_3 3^{10}$

Exponent, Sum and Difference Properties of Logarithms

Let b be a positive number, not equal to 1, and let u and v be positive numbers.

$$\log_b (uv) = \log_b u + \log_b v$$

$$\log_b \left(\frac{u}{v} \right) = \log_b u - \log_b v$$

$$\log_b x^m = m \log_b x$$

Ex 10: Use these properties to expand these expressions.

- a) $\log \sqrt{x^2(x+2)}$ b) $\ln \left(\frac{x^2-1}{x^3} \right), x > 1$

Ex 11: Use these properties to contract these expressions into a single term.

- a) $3 \log x + 4 \log y - 5 \log z$ b) $\frac{1}{2} [\ln(x+1) + 2 \ln(x-1)] - 6 \ln x$