

Math 1050 ~ College Algebra

16 Introduction to Exponentials and Logarithms

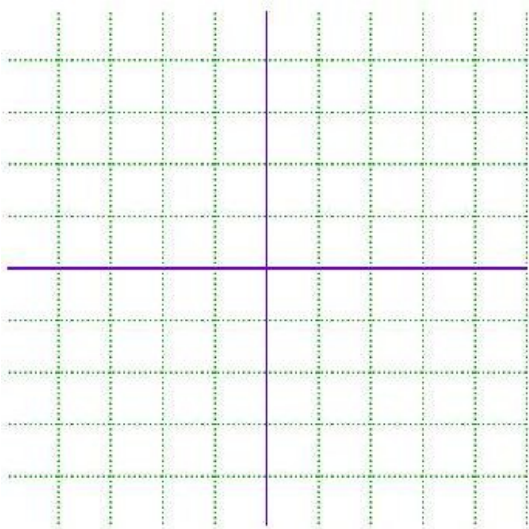
Definition of an Exponential Function

An exponential function is one in which the variable is in the exponent.

$$f(x) = b^x, \text{ where } b > 0, b \neq 1, x \in \mathcal{R}$$

EX 1

Fill out the table and plot the graph of $y = 2^x$.



x	$f(x)$	$(x, f(x))$
-3		
-1		
0		
1		
2		

Notice these things about the graph above.

Domain

Range

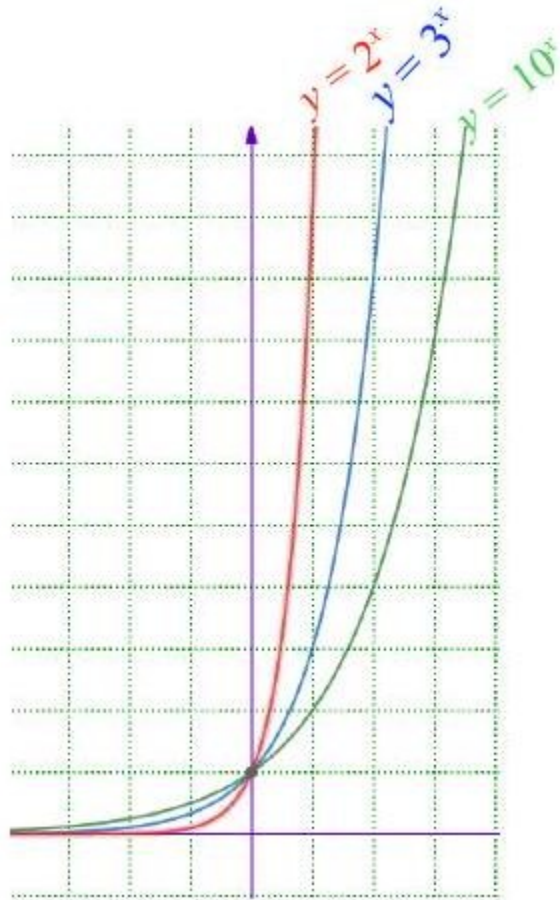
y-intercept

Horizontal asymptote

Exponential growth

Horizontal line test

As the base, b changes note how little else does.



We can use transformations learned previously to graph variations.

EX 2

Use transformations to sketch these functions.

$f(x) = 3^{(x-1)} + 2$	$g(x) = 2^{(3-x)} - 1$

Definition of a Logarithm

For $y > 0$ and b a positive constant other than 1, $\log_b y$ is called a logarithm in base b of y , and is the power of b that gives y .

$$y = \log_b x \Leftrightarrow x = b^y$$

EX 3

Find the exact value for each of these.

3a)

$$\log_2 16$$

3b)

$$\log_{10} 100000$$

3c)

$$\log_5 \frac{1}{125}$$

3d)

$$\log_8 4$$

EX 4

Convert from logarithmic form to exponential form or visa versa.

4a)

$$9^{3/2} = 27$$

4b)

$$\log_8 \sqrt{8} = \frac{1}{2}$$

4c)

$$\log_{32} 4 = \frac{2}{5}$$

4d)

$$10^{-3} = 0.001$$

To solve a logarithmic equation, it is convenient to turn it into an exponential equation.

EX 5

Solve each equation.

5a)

$$\log_2 (x - 1) = 5$$

5b)

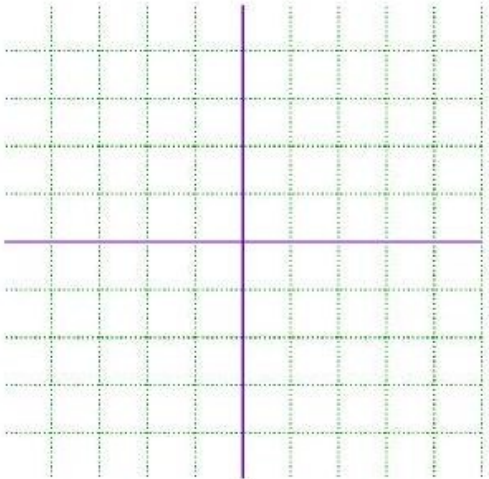
$$\log_{10} (3z) = 2$$

Definition of a Logarithmic Function

$f(x) = \log_b x$ is a logarithmic function with $x > 0$, $b > 0$ and $b \neq 1$.

EX 6

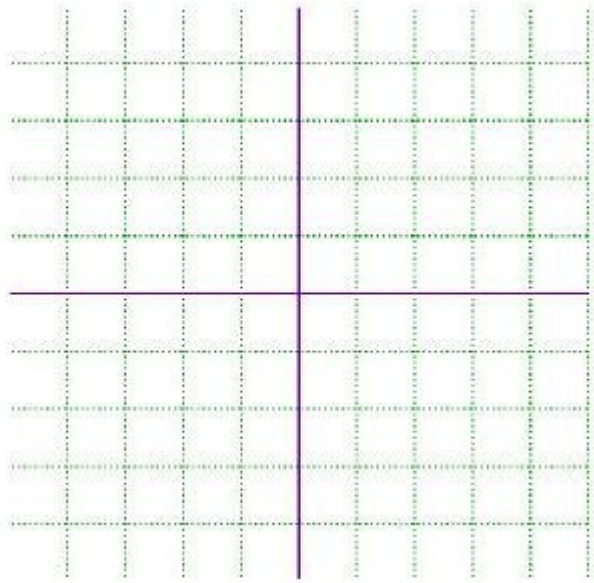
Fill in the table and sketch a graph of $f(x) = \log_2 x$



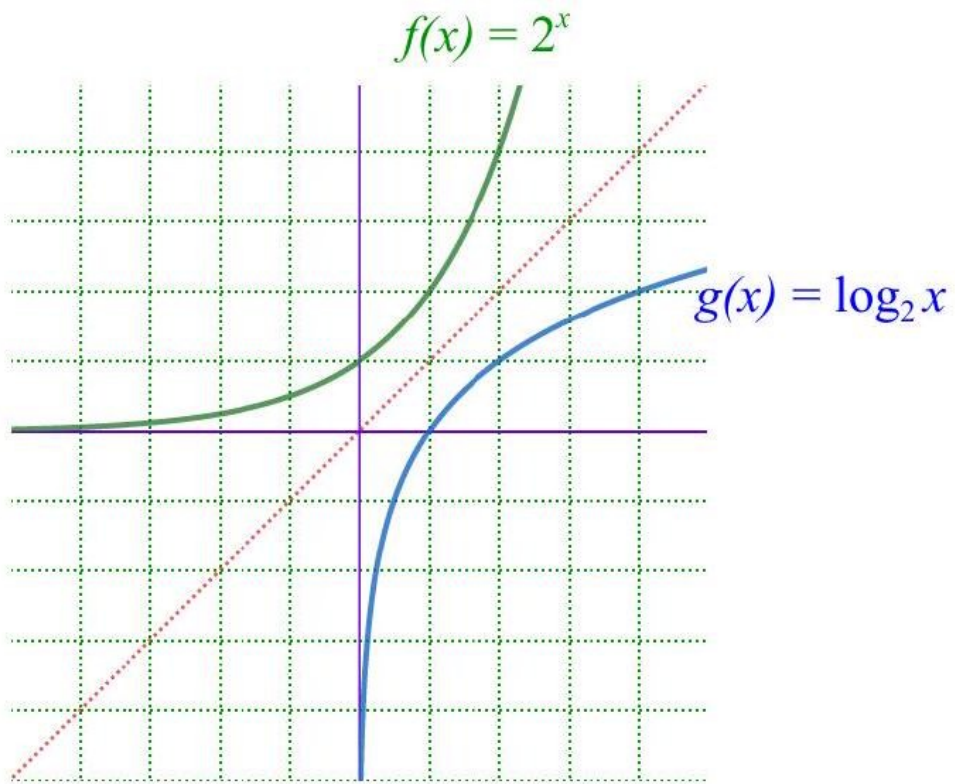
x	$f(x)$	$(x, f(x))$
$1/4$		
$1/2$		
1		
2		
4		

EX 7

Use transformations to sketch $f(x) = -\log_2(x) + 1$



Relationship of Exponential and Logarithmic Functions



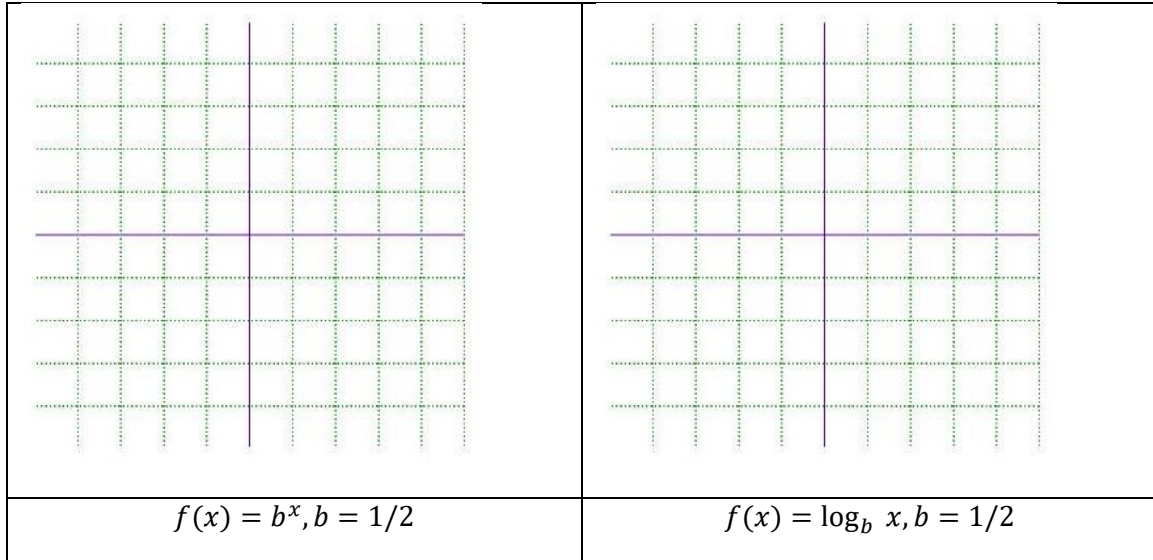
EX 7

Note the symmetry in the two functions. Compute this.

$$(g \circ f)(x) =$$

EX 8

All of the previous graphs given in this lesson have the characteristic that $b > 1$. Examine what happens when $0 < b < 1$. Sketch below for $b = 1/2$.



Properties of Graphs of Logarithmic And Exponential Functions