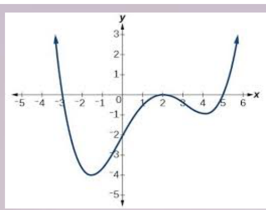
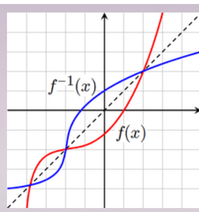
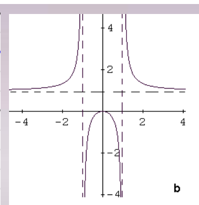
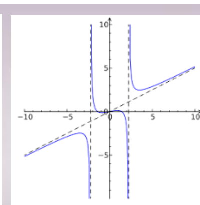
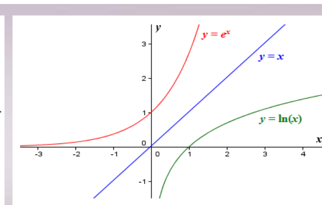










Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

14 Graphs with Holes and Variations on Asymptotes

Learning Objectives

- Identify holes in the graph of a rational function.
- Graph rational functions without vertical asymptotes.
- Find slant (oblique) asymptotes.
- Graph rational functions having slant asymptotes.

Since there can be no points on the vertical asymptotes, what happens in an example like this?

Note: To graph a rational fn, add to your steps to ① simplify completely and ② find all holes (before finding VA, etc.)

Ex 1: Analyze and graph.

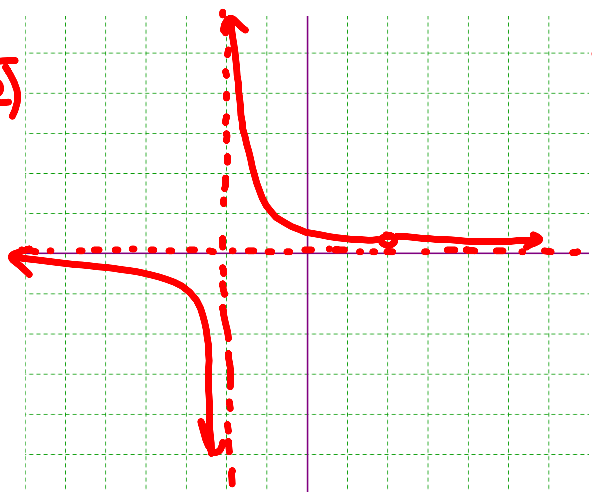
$$H(x)=\frac{x-2}{x^2-4}=\frac{x-2}{(x-2)(x+2)}$$

$H(x)=\frac{1}{x+2}$

we lost info that $x\neq 2$

hole: $(2,\frac{1}{4})$ $y=\frac{1}{2+2}$

VA: $x=-2$



HA: $y=0$

Graphing Rational Functions with No Vertical Asymptotes

Ex 2: Analyze and graph. $H(x) = \dfrac{2x+3}{x^2+2}$

VA: none

HA: $y = 0$

no holes

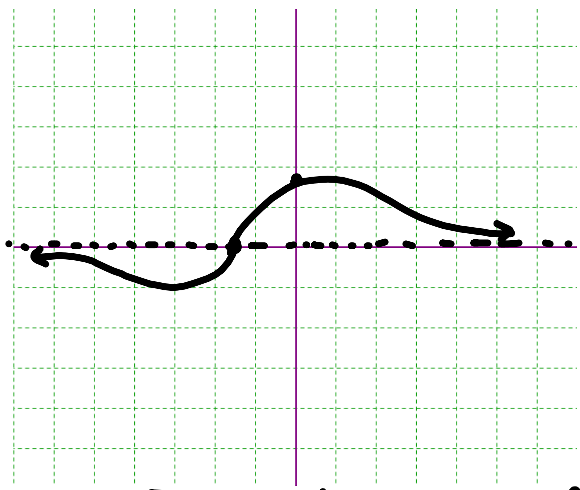
x-int: $0 = 2x + 3$

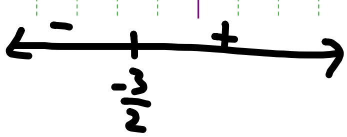
$x = \dfrac{-3}{2}$

$\boxed{\left(-\frac{3}{2}, 0\right)}$

y-int: $y = \dfrac{0+3}{0+2} = \dfrac{3}{2}$

$\boxed{\left(0, \frac{3}{2}\right)}$





sign line

Identifying Slant (Oblique) Asymptotes

SA

NOTE: these may or may not be lines. Slant asymptotes describe end behavior, which could be curvy.

Strategy to find SA:

(0) look for HA first.

If there is NO HA, then

(1) do long division (i.e. divide denominator of rational fn into its numerator)

(2) y = result of division WITHOUT remainder is SA.

NOTE: we only look for SA when degree of numerator > degree of denominator.

Ex 3: Analyze and graph.

$$H(x) = \frac{x^2 - x + 6}{x + 3}$$

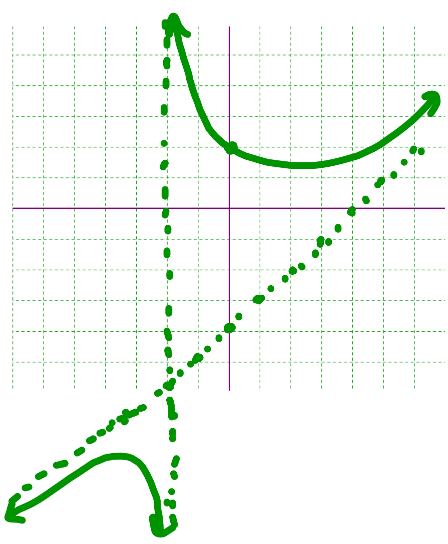
(numerator does not factor) $\Rightarrow$ no holes

VA: $x = -3$

HA: none

need to look for SA.

$$\begin{array}{r} x - 4 \\ x+3 \overline{\big) \cancel{x^2} - x + 6} \\ -(\cancel{x^2} + 3x) \\ \hline \cancel{-4x} + 6 \\ -(\cancel{-4x} - 12) \\ \hline 18 \end{array}$$



x-int:

$0 = x^2 - x + 6$

N.S.

$\boxed{\text{no x-int.}}$

y-int:

$y = \frac{6}{3} = 2$

$\boxed{(0,2)}$

$H(x) = x - 4 + \frac{18}{x+3}$

$\Rightarrow$ SA: $y = x - 4$

Ex 4: Analyze and graph.

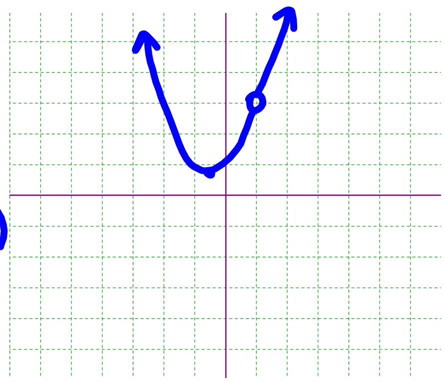
$$f(x)=\frac{x^3-1}{x-1}$$

$$f(x)=\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$$

$f(x)=x^2+x+1$

hole: $(1,3)$

$y=1^2+1+1=3$



no VA

no HA

vertex:

$x=\frac{-1}{2(1)}=\frac{-1}{2}$

$\left(-\frac{1}{2},\frac{3}{4}\right)$

$\left(\frac{-1}{2}\right)^2+\frac{-1}{2}+1=\frac{1}{4}+\frac{1}{2}$

$=\frac{3}{4}$