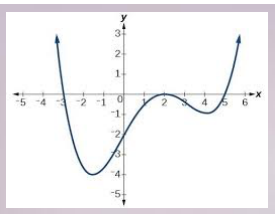
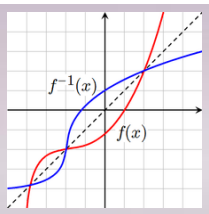
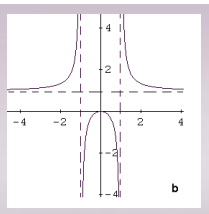
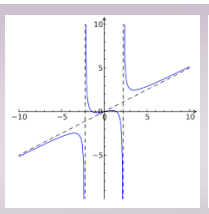
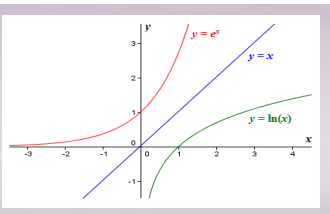





Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

13 Graphing Rational Functions

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Learning Objectives

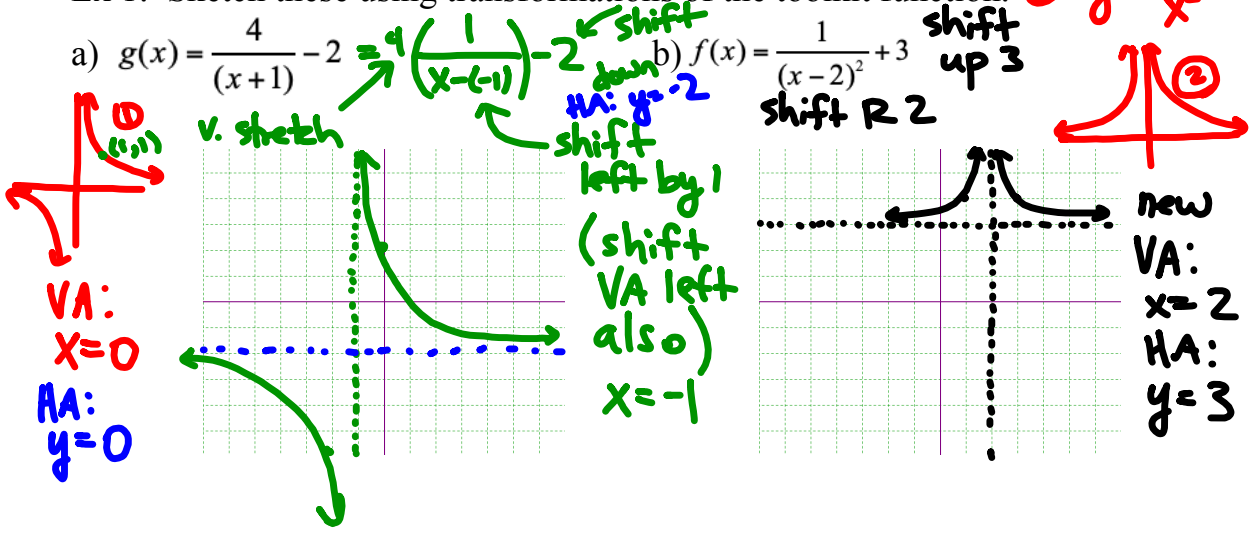
- Graph irreducible rational functions with denominators of degree greater than one and numerators having the same or a lesser degree.

In our toolkit of functions, we have two rational functions. ① $y=\frac{1}{x}$ and ② $y=\frac{1}{x^2}$

Ex 1: Sketch these using transformations of the toolkit function.

a) $g(x)=\dfrac{4}{(x+1)}-2$ $=4\left(\dfrac{1}{x-(-1)}\right)-2$ ← shift down 2; HA: $y=-2$; shift left by 1 (shift VA left also) $x=-1$

b) $f(x)=\dfrac{1}{(x-2)^2}+3$ shift up 3; shift R 2



Not all rational functions can be put in this form. It is helpful to follow the steps in the previous lesson to get a graph of a rational function.

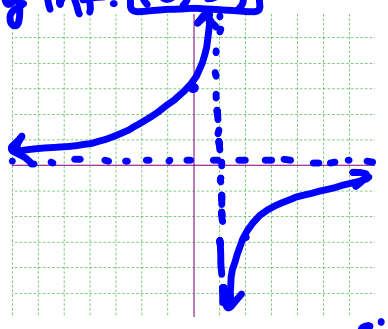
1. Determine the domain and plot vertical asymptotes.
2. Find and plot the x- and y- intercepts.
3. Determine and plot the end-behavior asymptotes.
4. Use a sign-line and the value of other points to complete the graph.

Ex 2: For each of these, determine the x and y-intercepts, vertical and horizontal asymptotes and sketch a graph.

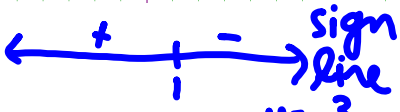
a) $f(x)=\frac{3}{1-x}$

VA: x=1
HA: y=0
N.S.

x-int: $0=\frac{3}{1-x}$
no x-int
y-int: (0,3)



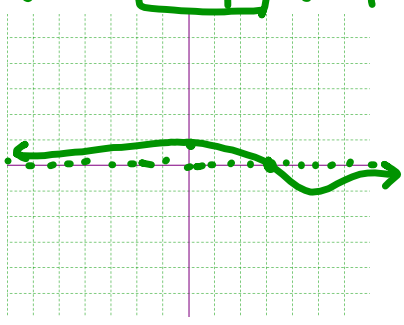
sign line



b) $g(x)=\frac{3-x}{x^2+4}$

VA: none
HA: y=0

x-int: (3,0)
$0=3-x \Rightarrow x=3$
y-int: $(0,\frac{3}{4})$ $y=\frac{3}{4}$



y-int: $(0,\frac{3}{2})$

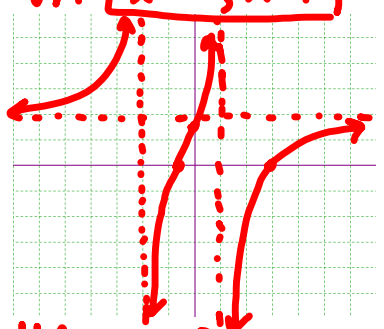
$y=\frac{2(0^2)-5(0)-3}{0^2+0-2}$

$y=\frac{-3}{-2}=\frac{3}{2}$

c) $h(x)=\frac{2x^2-5x-3}{x^2+x-2}$

$h(x)=\frac{(2x+1)(x-3)}{(x+2)(x-1)}$

VA: x=-2, x=1



HA: $y=\frac{2}{1}=2$
y=2

x-int: set y=0

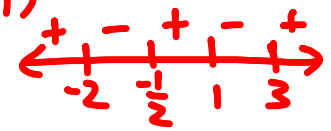
$(2x+1)(x-3)=0$

$x=-\frac{1}{2}, 3$

$(-\frac{1}{2},0), (3,0)$

$h(x)=\frac{(2x+1)(x-3)}{(x+2)(x-1)}$

sign line



x=-1000	x=-1	x=2	x=1000
$\frac{-(-)}{-(-)}$	$\frac{-(-)}{+(-)}$	$\frac{+(-)}{+(+)}$	$\frac{+(+)}{+(+)}$

Ex 3: Analyze and graph.

$$f(x)=\frac{(x-4)(x-2)^2}{(x+3)^2(x-1)}$$

VA: $x=1, x=-3$

HA: end behavior $y=1$

is like $\frac{x(x^3)}{x^2(x)}=\frac{x^3}{x^3}=1$

x-int: $0=(x-4)(x-2)^2$

$x=4,2$

$(4,0)$ $(2,0)$

y-int: $y=\frac{(0-4)(-2)^2}{3^2(-1)}=\frac{-16}{-9}=\frac{16}{9}$

$\left(0,\frac{16}{9}\right)$