

Math 1050 ~ College Algebra

12 Introduction to Rational Functions

A rational function is a ratio of two polynomial functions. $f(x) = \frac{N(x)}{D(x)}$ where $N(x)$ and $D(x)$ are polynomials.

EX 1

Determine which of these functions are rational functions.

1a)

$$f(x) = \frac{x^2+1}{x+4}$$

1b)

$$f(x) = \frac{3x+2}{\sqrt{x}-3}$$

1c)

$$f(x) = \frac{x^2 - 2x - 3}{\pi}$$

1d)

$$f(x) = \frac{x^{2.5} + 5}{x^2 - 25}$$

Vertical Asymptotes of Simplified Rational Functions

- determined by finding disallowed denominator values
- line $x = a$ where $D(a) = 0$
- graph will never cross or touch

EX 2

Find the domain and the vertical asymptotes for these functions.

2a)

$$f(x) = \frac{2x^2}{x^2 - 1}$$

2b)

$$f(x) = \frac{3x}{x^2 + 1}$$

2c)

$$f(x) = \frac{x + 4}{4x - 2x^2}$$

Horizontal Asymptotes

- end behavior of the graph
- line $y = b$ where $\lim_{x \rightarrow \pm\infty} f(x) = b$
- graph may cross it
- depends on degree of $N(x)$ and $D(x)$
- degree ($N(x)$) < degree ($D(x)$), $y = 0$
- degree ($N(x)$) = degree ($D(x)$), $y =$ ratio of the leading coefficients.

EX 3

Find the horizontal asymptotes of these functions.

3a)

$$f(x) = \frac{2x^2}{x^2 - 1}$$

3b)

$$f(x) = \frac{3x}{x^2 + 1}$$

3c)

$$f(x) = \frac{x + 4}{4x - 2x^2}$$

EX 4

For each of these functions, determine the x and y -intercepts, vertical and horizontal asymptotes and draw a quick sketch.

4a)

$$f(x) = \frac{2x^2}{x^2 - 1}$$

4b)

$$f(x) = \frac{3x}{x^2 + 1}$$

4c)

$$f(x) = \frac{x + 4}{4x - 2x^2}$$

