Math 1050 ~ College Algebra

10 Complex Zeros of Polynomials

Complex Numbers

The imaginary unit i satisfies the following properties:

- $i^2 = -1$
- If c is a real number, $c \geq 0$ then $\sqrt{-c} = (\sqrt{c}) \cdot i$

A complex number is a number of the form $a + bi$ where a and b are real numbers and i is the imaginary unit.

The real numbers are a subset of the complex numbers.

The conjugate of a complex number, $a + bi$ is $a - bi$.

Expressed in symbols, $\overline{a + bi} = a - bi$.

EX 1

Identify a, b and the conjugate of each of these complex numbers.

1a)

$-2 + 5\text{i}$

1b)

$6i$

1c)

53

1d)

$\pi - i$

Arithmetic on these numbers is as expected.

EX 2

Perform these operations on complex numbers.

2a)

$(1 - 3i) + (2 + 5i)$

2b)

$(1 - 3i)(2 + 5i)$

2c)

$(1 - 3i) - (2 + 5i)$

2d)

$\frac{1-3i}{2+5i}$

2e)

$\sqrt{-3}\sqrt{-12}$

2f)

$\sqrt{(-3)(-12)}$

EX 3

Perform this multiplication.

$$(x - (1 + 2i))(x - (1 - 2i))$$

Complex Roots of Polynomial Functions

The Fundamental Theorem of Algebra and Complex Factorization.
If f is a polynomial function with degree $n \geq 1$:

- f has at least one complex zero.
- In actuality, f has exactly n zeros, counting multiplicities.
- f has precisely n factors.

Furthermore:

- Complex zeros occur in conjugate pairs.
- Every polynomial can be factored into linear and quadratic factors with real coefficients.

EX 4

Determine the complex zeros of $f(x) = 3x^2 - 2x + 2$.

EX 5

Given $x + 3i$ is a factor of f(x) $= 2x^3 - 11x^2 + 18x - 99$, find all other zeros.

EX 6

Use the techniques in this section and the last to find all the zeros of

$f(x) = x^5 + 6x^4 + 10x^3 + 6x^2 + 9x.$