MATH 1010 ~ Intermediate Algebra

Chapter 9: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Section 9.5: Solving Exponential and Logarithmic Equations

Objectives:

- Solve basic exponential and logarithmic equations.
- Use inverse properties to solve exponential and logarithmic equations.

$$\log_2(x-2)=\log_2 x+3$$

$$500e^{-0.2x}=100$$

Solve

1) $9^{x+3} = 9^{10}$

(exponential because variable in exponent)

$x+3=10$

$x=7$

2) $\log_3(4-3x) = \log_3(2x+9)$

logarithmic eqn because variable inside log fn

$4-3x=2x+9$

$4=5x+9$

$-5=5x \Leftrightarrow x=-1$

Notes:

- exponential eqns – do not need to check answers
- logarithmic eqns – it IS necessary to check answers (because we can only take log of a positive #)

3) $\dfrac{6e^{-x}}{6} = \dfrac{3}{6}$

$e^{-x} = \frac{1}{2}$

2(a)

$\ln \frac{1}{2} = -x$

$-\ln \frac{1}{2} = x$

2(b)

$\ln e^{-x} = \ln \frac{1}{2}$

$-x \ln e = \ln \frac{1}{2}$

$-x = \ln \frac{1}{2}$

$x = -\ln \frac{1}{2}$

note: $x = \ln\left(\frac{1}{2}\right)^{-1} = \boxed{\ln 2}$

Strategy to solve exp. eqn

1. Isolate exponential term.
2. (a) use defn log to rewrite the eqn; OR (b) take log of both sides (choose appropriate base for log)
3. finish solving

4) $50(3-e^{2x}) = 125$

$\frac{50(3-e^{2x})}{50} = \frac{125}{50}$

$3 - e^{2x} = \frac{5}{2}$

$\frac{-e^{2x}}{-1} = \frac{\frac{5}{2}-3}{-1}$

$e^{2x} = \frac{1}{2}$

defn of log

$\frac{\ln\frac{1}{2}}{2} = \frac{2x}{2}$

$\boxed{\frac{1}{2}\ln\left(\frac{1}{2}\right) = x}$

$x = \ln\sqrt{\tfrac{1}{2}} = -\ln\sqrt{2}$

5) $\frac{500}{1+e^{-0.1x}} = 400$

$500 = 400(1+e^{-0.1x})$

$\frac{5}{4} = 1 + e^{-0.1x}$

$-1 \quad -1$

$\frac{1}{4} = e^{-0.1x}$

$\ln\frac{1}{4} = \ln e^{-0.1x}$

$\ln\left(\frac{1}{4}\right) = -0.1x$

$\boxed{\frac{-1}{0.1}\ln\left(\frac{1}{4}\right) = x} = -10\ln\left(\frac{1}{4}\right)$

$= 10\ln 4$

6) $\frac{3}{2}\cdot\frac{2}{3}\log_3(x+1) = -1\cdot\frac{3}{2}$

$\log_3(x+1) = \frac{-3}{2}$

$3^{-3/2} = x+1$

$-1 \quad -1$

$x = 3^{-3/2} - 1$ ✓

$= \frac{1}{\sqrt{3^3}} - 1$

$= \frac{1}{3\sqrt{3}} - 1$

Strategy for solving logarithmic eqn

1. use log properties to condense log terms completely
2. use defn of log to rewrite eqn in exp. form
3. finish solving
4. check answer

7) $\log_3(x-2)+\log_3 5=3$

$\log_3(5(x-2))=3$

$3^3=5(x-2)$

$27=5x-10$

$37=5x$

$x=\frac{37}{5}$ or $7\frac{2}{5}$

8) $\log_3(2x)+\log_3(x-1)-\log_3 4=1$

$\log_3(2x(x-1))-\log_3 4=1$

$\log_3\left(\frac{2x(x-1)}{4}\right)=1$

$4\cdot 3^1=\frac{2x(x-1)}{4}\cdot 4$

$12=2x(x-1)$

$12=2x^2-2x$

$0=2x^2-2x-12$

$0=2(x^2-x-6)$

$\frac{0}{2}=\frac{2(x-3)(x+2)}{2}$

$0=(x-3)(x+2)$

$x-3=0$ $\quad$ $x+2=0$

$x=3$ $\quad$ $x=-2$

$x=3,-2$

if $x=-2$,

$\log_3(-4)$ DNE

throw away

$x=-2$

soln: $x=3$

Applications

1) At what interest rate (compounded continuously) will you have to invest $10,000 to make sure it doubles in ten years?

$y = Pe^{rt}$

P = principal
t = time (yrs)
r = interest rate
y = value of acct. after t years

$P = 10000 \quad r = ? \quad t = 10$

$y = 20000$

$20000 = 10000e^{10r}$

$2 = e^{10r}$

$\ln 2 = 10r$

$\rightarrow r = \frac{1}{10}\ln 2 \simeq 0.0693$

6.93%

2) How long will it take a bacteria culture of 200 mg to grow to 51,200 mg if it doubles every hour?

n = # hours

y = bacteria amt (mg)

n	y
0	200
1	2(200) = 400 = $2^1(200)$
2	2(400) = 800 = $2(2^1(200)) = 2^2(200)$
3	2(800) = 1600 = $2^3(200)$
4	2(1600) = 3200 = $2^4(200)$
5	2(3200) = 6400 = $2^5(200)$
6	2(6400) = 12,800
7	2(12,800) = 25,600
8	2(25,600) = 51,200

after 8 hrs.

general

$y = 200(2^n)$

$51200 = 200(2^n)$

$256 = 2^n$

$256 = 2^8$

$n = 8$