

Section 7.5: Solving Radical Equations

Objective:

- * Solve a radical equation.

$$\sqrt{2x-1} - \sqrt{3x} = 2$$

TRUE or FALSE?

If $u = v$, then $u^n = v^n$.

True

Strategy for Solving Radical Egn

- ① Isolate one of the radical expressions on one side of egn.
- ② Raise both sides of egn to the power that will undo the root.
- ③ Finish solving.

① EXAMPLE

Solve for the variable.

WARNING: check answers for all even root egn's.

$$a) \sqrt{x} - 5 = 0$$

$$\begin{aligned} (\sqrt{x})^2 &= (5)^2 \\ x &= 25 \end{aligned}$$

$$b) \sqrt{x^2 + 5} - 3 = 0$$

$$\begin{aligned} (\sqrt{x^2 + 5})^2 &= (3)^2 \\ x^2 + 5 &= 9 \\ x^2 &= 4 \end{aligned} \quad \begin{aligned} x-4 &= 0 \\ (x-2)(x+2) &= 0 \\ x-2 &= 0 \quad \text{or} \quad x+2 = 0 \\ x &= 2 \quad , \quad x = -2 \end{aligned}$$

$$c) \sqrt{3y+1} - \sqrt{y+15} = 0$$

$$\begin{aligned} (\sqrt{3y+1})^2 &= (\sqrt{y+15})^2 \\ 3y+1 &= y+15 \end{aligned} \quad \begin{aligned} 2y+1 &= 15 \\ 2y &= 14 \\ y &= 7 \end{aligned}$$

$$d) \sqrt[3]{10-3x} = \sqrt[3]{2-x}$$

$$\begin{aligned} 2^3(10-3x) &= 2-x \\ \equiv & \end{aligned}$$

$$8(10-3x) = 2-x$$

$$80-24x = 2-x$$

$$80 = 2 + 23x$$

$$78 = 23x$$

$$x = 78/23$$

② EXAMPLE

Solve for the variable.

$$\text{check: } \sqrt{\frac{13}{4} + 3} - \sqrt{\frac{13}{4} - 1} = \sqrt{\frac{25}{4}} - \sqrt{\frac{9}{4}}$$

$$= \frac{5}{2} - \frac{3}{2} = 1 \quad \checkmark$$

$$a) \quad \sqrt{x+3} - \sqrt{x-1} = 1$$

$$+ \sqrt{x-1} \quad + \sqrt{x-1}$$

$$(\sqrt{x+3})^2 = (1 + \sqrt{x-1})^2$$

$$x+3 = (1 + \sqrt{x-1})(1 + \sqrt{x-1})$$

$$x+3 = 1 + \cancel{\sqrt{x-1}} + \cancel{\sqrt{x-1}} + x - 1$$

$$\begin{matrix} x \\ -x \end{matrix} + 3 = \begin{matrix} 2\sqrt{x-1} \\ -x \end{matrix} + x$$

$$\frac{3}{2} = \frac{2\sqrt{x-1}}{2}$$

$$\left(\frac{3}{2}\right)^2 = (\sqrt{x-1})^2$$

$$\begin{matrix} \frac{9}{4} \\ +1 \end{matrix} = \begin{matrix} x-1 \\ +1 \end{matrix}$$

$$\boxed{\frac{13}{4} = x}$$

check:

$$\begin{aligned} \sqrt{\frac{13}{4}} + \sqrt{\frac{13}{4} + 2} &= \frac{1}{2} + \sqrt{\frac{9}{4}} \\ &= \frac{1}{2} + \frac{3}{2} = 2 \quad \checkmark \end{aligned}$$

$$b) \quad \begin{matrix} \sqrt{x} \\ -\sqrt{x} \end{matrix} + \begin{matrix} \sqrt{x+2} \\ -\sqrt{x} \end{matrix} = 2$$

$$(\sqrt{x+2})^2 = (2 - \sqrt{x})^2$$

$$x+2 = (2 - \sqrt{x})(2 - \sqrt{x})$$

$$x+2 = 4 - 2\sqrt{x} - 2\sqrt{x} + x$$

$$\begin{matrix} x+2 \\ -x \end{matrix} = \begin{matrix} 4 \\ -4 \end{matrix} + \begin{matrix} x \\ -x \end{matrix} - 4\sqrt{x}$$

$$\frac{-2}{-4} = \frac{-4\sqrt{x}}{-4}$$

$$\left(\frac{1}{2}\right)^2 = (\sqrt{x})^2$$

$$\boxed{\frac{1}{4} = x}$$

③ EXAMPLE

Solve for the variable.

a) $(x+4)^{5/3} = 32$

$$\left((x+4)^{5/3} \right)^{3/5} = 32^{3/5}$$

$$x+4 = 32^{3/5}$$

$$x+4 = \sqrt[5]{32^3}$$

$$x+4 = (\sqrt[5]{32})^3$$

$$x+4 = 2^3$$

$$x+4 = 8$$

$$x = 4$$

check: $(x+4)^{5/3} = 32$

if $x=4$, $(8)^{5/3} = (\sqrt[3]{8})^5$
 $= 2^5 = 32 \checkmark$

b) $\sqrt{x} = x - 6$

$$(\sqrt{x})^2 = (x-6)^2$$

$$x = (x-6)(x-6)$$

$$x = x^2 - 6x - 6x + 36$$

$$x = x^2 - 12x + 36$$

$$0 = x^2 - 13x + 36$$

$$0 = (x-4)(x-9)$$

$$x-4=0 \text{ or } x-9=0$$

$$x=4 \text{ or } x=9$$

check $\sqrt{x} = x - 6$

~~x=4~~: $\sqrt{4} \stackrel{?}{=} 4-6$

~~2~~ $\stackrel{?}{=} -2$ NO

x=9: $\sqrt{9} \stackrel{?}{=} 9-6$

$3 \stackrel{?}{=} 3 \checkmark$