MATH 1010 ~ Intermediate Algebra

Chapter 7: RADICALS AND COMPLEX NUMBERS

Section 7.4: Multiplying and Dividing Radical Expressions

Objectives:

- ✯ Use the distributive property to multiply radical expressions.
- ✯ Determine the product of conjugates.
- ✯ Simplify quotients involving radicals by rationalizing the denominators.

$$(5\sqrt{x^3})(-x\sqrt{4x}) \div (3x\sqrt{x})$$

Rule for Multiplying and Dividing Radical Expressions

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

and

$$\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$$

ex $\sqrt{3}\sqrt{5}$

$= \sqrt{15}$

Roots (radicals) distribute through multiplication & division

WARNING: Roots never distribute through addition or subtraction

(ex $\sqrt{1+3} \neq \sqrt{1} + \sqrt{3}$)

① EXAMPLE

Multiply and simplify.

a) $\sqrt{6}\cdot\sqrt{2} = \sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

b) $\sqrt[3]{6}\cdot\sqrt[3]{16} = \sqrt[3]{(2\cdot3)(2\cdot2\cdot2\cdot2)} = \sqrt[3]{12}\,\sqrt[3]{2^3}$

$= \boxed{2\sqrt[3]{12}}$

c) $\sqrt{5}\left(2+\sqrt{3}\right) = 2\sqrt{5}+\sqrt{5}\sqrt{3}$

$= \boxed{2\sqrt{5}+\sqrt{15}}$

d) $\sqrt{6}\left(\sqrt{12}-\sqrt{3}\right)$

$= \sqrt{6}\left(\sqrt{4}\sqrt{3}-\sqrt{3}\right)$

$= \sqrt{6}\left(2\sqrt{3}-\sqrt{3}\right)$

$= \sqrt{6}\left(\sqrt{3}\right)$

$= \sqrt{6\cdot3} = \sqrt{2\cdot3^2}$

$= \sqrt{2}\sqrt{3^2}$ (or $\sqrt{2}(3)$)

$= \boxed{3\sqrt{2}}$

② EXAMPLE

Perform the indicated operation and simplify the answer.

a) $\left(2\sqrt{7}-3\right)\left(\sqrt{7}+2\right) = 2\sqrt{7}\sqrt{7} + 2\cdot 2\sqrt{7} - 3\sqrt{7} - 6$

$= 2(7) + 4\sqrt{7} - 3\sqrt{7} - 6$

$= 14 + \sqrt{7} - 6 = \boxed{8+\sqrt{7}}$

b) $\left(2-\sqrt{x}\right)\left(1+\sqrt{x}\right)$ (note: $x \geq 0$ $\Rightarrow \sqrt{x^2} = x$)

$= 2 + 2\sqrt{x} - \sqrt{x} - \sqrt{x^2}$

$= \boxed{2+\sqrt{x}-x}$

c) $\left(3-\sqrt{x}\right)\left(3+\sqrt{x}\right)$ (note: $x \geq 0 \Rightarrow \sqrt{x^2} = x$)

$= 9 + \cancel{3\sqrt{x}} - \cancel{3\sqrt{x}} - \sqrt{x^2}$

$= \boxed{9-x}$

The conjugate of $\sqrt{a}+\sqrt{b}$ *is* $\sqrt{a}-\sqrt{b}$

The conjugate of $\sqrt{a}-\sqrt{b}$ *is* $\sqrt{a}+\sqrt{b}$

note: $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$

$= a - \cancel{\sqrt{ab}} + \cancel{\sqrt{ab}} - \sqrt{b^2}$

$= a - b$

③ EXAMPLE

Determine the conjugate of each expression and multiply the expression by it.

a) $2+\sqrt{7}$ conjugate: $2-\sqrt{7}$

$(2+\sqrt{7})(2-\sqrt{7}) = 4 - \cancel{2\sqrt{7}} + \cancel{2\sqrt{7}} - \sqrt{7^2}$

$= 4 - 7 = -3$

b) $3-\sqrt{5}$ conjugate: $3+\sqrt{5}$

$(3-\sqrt{5})(3+\sqrt{5}) = 9 + \cancel{3\sqrt{5}} - \cancel{3\sqrt{5}} - \sqrt{5^2}$

$= 9 - 5 = 4$

c) $2\sqrt{3}+\sqrt{x}$

conjugate: $2\sqrt{3}-\sqrt{x}$

$(2\sqrt{3}+\sqrt{x})(2\sqrt{3}-\sqrt{x})$

$= 4(3) - \cancel{2\sqrt{3}\sqrt{x}} + \cancel{2\sqrt{3}\sqrt{x}} - x$

$= 12 - x$

④ EXAMPLE

Rationalize the denominators and simplify.

$$a)\ \left(\frac{\sqrt{3}}{1-\sqrt{5}}\right)\overset{\text{try}}{\left(\frac{\sqrt{5}}{\sqrt{5}}\right)} = \frac{\sqrt{15}}{\sqrt{5}-5}$$ doesn't work

$$\left(\frac{\sqrt{3}}{1-\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{1+\sqrt{5}}\right) = \frac{\sqrt{3}(1+\sqrt{5})}{1+\cancel{\sqrt{5}}-\cancel{\sqrt{5}}-\sqrt{25}} = \frac{\sqrt{3}(1+\sqrt{5})}{1-5}$$

$$= \boxed{\frac{\sqrt{3}(1+\sqrt{5})}{-4}}$$

$$b)\quad 7 \div (x-\sqrt{3})$$

$$= \left(\frac{7}{x-\sqrt{3}}\right)\left(\frac{x+\sqrt{3}}{x+\sqrt{3}}\right) = \frac{7(x+\sqrt{3})}{x^2+\cancel{\sqrt{3}x}-\cancel{\sqrt{3}x}-3}$$

$$\boxed{\sqrt{3}\sqrt{3}=3}$$

$$= \boxed{\frac{7(x+\sqrt{3})}{x^2-3}}$$

$$c)\quad \frac{5\sqrt{2}}{3\sqrt{2}+\sqrt{6}}$$

$$= \frac{5\sqrt{2}}{3\sqrt{2}+\sqrt{2}\sqrt{3}} = \frac{5\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}(3+\sqrt{3})} = \frac{5}{(3+\sqrt{3})}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right) = \frac{5(3-\sqrt{3})}{9-\cancel{3\sqrt{3}}+\cancel{3\sqrt{3}}-3}$$

$$= \boxed{\frac{5(3-\sqrt{3})}{6}}$$

$$d)\ \left(\frac{2-\sqrt{3}}{\sqrt{2}+\sqrt{7}}\right)\left(\frac{\sqrt{2}-\sqrt{7}}{\sqrt{2}-\sqrt{7}}\right)$$

$$= \frac{2\sqrt{2}-2\sqrt{7}-\sqrt{6}+\sqrt{21}}{2-\cancel{\sqrt{14}}+\cancel{\sqrt{14}}-7} = \boxed{\frac{2\sqrt{2}-2\sqrt{7}-\sqrt{6}+\sqrt{21}}{-5}}$$