MATH 1010 ~ Intermediate Algebra

Chapter 7: RADICALS AND COMPLEX NUMBERS

Section 7.1: Radicals and Rational Exponents

Objectives:

- Determine the nth roots of numbers and evaluate radical expressions.
- Use the rules of exponents to evaluate or simplify expressions with rational exponents.
- Evaluate radical functions and find the domain of radical functions.

$64^{2/3}$

$-64^{3/2}$

$(-64)^{2/3}$

$64^{3/2}$

n^th root

The principal n^th root of *a* has the same sign as *a*.

ex $\sqrt{4} = 2$

$\sqrt[4]{1} = 1$

$$a = b^n \Leftrightarrow b = \sqrt[n]{a}$$

Notation

$$\boxed{\sqrt[n]{a} = a^{1/n}}$$

"n^th root of a" equals "a to the 1 over n power"

$$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} = \left(a^m\right)^{1/n} = a^{m/n}$$

$$= \left(a^{1/n}\right)^m$$

ex $27^{2/3} = \sqrt[3]{27^2} = \left(\sqrt[3]{27}\right)^2 = \left(27^2\right)^{1/3}$

$(3)^2 = 9$

① EXAMPLE

$a)\ \sqrt{36} =$ 6

$b)\ -\sqrt{36} =$

-6

$c)\ \sqrt{-25} =$ undefined

$d)\ \sqrt[3]{-8} = -2$

$(-2 \cdot -2 \cdot -2 = -8)$

$e)\ \sqrt[3]{27} = 3$

$(3 \cdot 3 \cdot 3 = 27)$

$f)\ \sqrt[3]{-27} = -3$

$(-3 \cdot -3 \cdot -3 = -27)$

Note: recommend memorizing squares up through 12, cubes up through 6

② EXAMPLE

a) $8^{4/3} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$

or $(2^3)^{4/3} = 2^{3 \cdot \frac{4}{3}} = 2^4 = 16$

b) $27^{-2/3} = \frac{1}{27^{2/3}} = \frac{1}{(3^3)^{2/3}} = \frac{1}{3^2} = \frac{1}{9}$

c) $\left(\frac{64}{125}\right)^{2/3} = \left(\sqrt[3]{\frac{64}{125}}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

d) $(-9)^{1/2} = \sqrt{-9}$

undefined

e) $-9^{1/2} = -\sqrt{9}$

$= -(3)$

$= -3$

③ EXAMPLE

Rewrite these with rational exponents.

a) $x\sqrt[4]{x^3} = x \quad \sqrt[4]{x^3} = x^1 x^{3/4} = x^{1+\frac{3}{4}} = x^{7/4}$

b) $\dfrac{\sqrt[3]{x^4}}{\sqrt{x^5}} = \dfrac{x^{4/3}}{x^{5/2}} = x^{\frac{4}{3}-\frac{5}{2}}$

$= x^{-7/6}$

$\frac{4}{3} - \frac{5}{2}$

$= \frac{8}{6} - \frac{15}{6}$

$= -7/6$

c) $\sqrt{\sqrt[3]{x}} = \sqrt{x^{1/3}} = \left(x^{1/3}\right)^{1/2}$

$= x^{\frac{1}{3}\left(\frac{1}{2}\right)} = x^{1/6}$

④ EXAMPLE

Simplify this.

$$\frac{(3x-2)^{5/3}}{\sqrt[3]{3x-2}} = \frac{(3x-2)^{5/3}}{(3x-2)^{1/3}} = (3x-2)^{5/3-1/3}$$

$$= (3x-2)^{4/3}$$

⑤ EXAMPLE

Determine the domain.

restrictions: can't take even root of negative #

$a)\ f(x)=\sqrt{x}$

domain: $x \geq 0$

$b)\ f(x)=\sqrt{x^4}$

x^4 is always nonnegative

$\sqrt{x^4}$ then is always okay

domain: $x \in \mathbb{R}$

$c)\ g(x)=\sqrt[3]{x}$

domain: $x \in \mathbb{R}$

$d)\ g(x)=\sqrt{x^3}$

note: x^3 can be negative

force: $x^3 \geq 0$

domain: $x \geq 0$