MATH 1010 ~ Intermediate Algebra

Chapter 5: POLYNOMIALS AND FACTORING

Section 5.4: Factoring by Grouping and Special Forms

Objectives:

- ✦ Factor the greatest common monomial factors from polynomials.
- ✦ Factor polynomials by grouping.
- ✦ Factor the difference of two squares.
- ✦ Factor the sum and difference of two cubes.
- ✦ Factor polynomials completely.

$a^3 - b^3 = ?$

$x^2 + 2xy + y^2 = ?$

$a^2 + b^2 = ?$

GCF: Greatest Common Factor

the largest number (or expression) that goes into all the given terms

Find the GCF.

a) $5x^4, \quad 20x^3, \quad 15x^5$

$5 \quad x^3 \quad x \qquad 5 \quad 4 \quad x^3 \qquad 3 \quad 5 \quad x^3 \quad x^2$

GCF $= 5x^3$

b) $21a^4b, \quad 12a^2b, \quad 15a^5b$

$\underline{3} \quad 7 \quad \underline{a^2} \quad a^2 \quad \underline{b} \qquad \underline{3} \quad 5 \quad \underline{a^2} \quad a^3 \quad \underline{b}$

$\underline{3} \quad 4 \quad \underline{a^2} \quad \underline{b}$

GCF $= 3a^2b$

① EXAMPLE:

Factor out the greatest common factor.

a) $24x^3 - 32x^2 = 8x^2(3x-4)$

(distributive property "backwards")

b) $4x^2(3x-1) - 6(3x-1)$

$= (3x-1)(4x^2-6)$

$= (3x-1)(2)(2x^2-3)$ or $2(3x-1)(2x^2-3)$

c) $x^3 - 5x^2 + x - 5$

grouping

$= x^2(x-5)+1(x-5)$

$=(x-5)(x^2+1)$

d) $(3x+7)(2x-1)+(x-6)(2x-1)$

$= (2x-1)\big((3x+7)+(x-6)\big)$

$= (2x-1)(3x+7+x-6)$

$= (2x-1)(4x+1)$

Difference of squares

$$\boxed{a^2 - b^2 = (a-b)(a+b)}$$

② EXAMPLE

Factor these

$$(a-b)(a+b) = a^2 \cancel{+ab} \cancel{-ab} - b^2$$
$$= a^2 - b^2$$

a) $9x^2 - 25$

$$= (3x)^2 - 5^2 = (3x-5)(3x+5)$$

$a = 3x$, $5 = b$

b) $a^2 - \frac{1}{16} = a^2 - \left(\frac{1}{4}\right)^2 = \left(a - \frac{1}{4}\right)\left(a + \frac{1}{4}\right)$

or $\left(a + \frac{1}{4}\right)\left(a - \frac{1}{4}\right)$

c) $(x+3)^2 - 49$

$a = x+3$
$b = 7$

$$= (x+3)^2 - 7^2$$
$$= (x+3+7)(x+3-7)$$
$$= (x+10)(x-4)$$

Sum and Difference of Cubes

$$\boxed{u^3 + v^3 = (u+v)(u^2 - uv + v^2)}$$

(check: $u^3 - \cancel{u^2 v} + \cancel{uv^2} + \cancel{u^2 v} - \cancel{uv^2} + v^3 = u^3 + v^3$)

$$\boxed{u^3 - v^3 = (u-v)(u^2 + uv + v^2)}$$

③ Example

Factor these.

a) $x^3 - 64$

$= x^3 - 4^3$ $\quad (u = x,\ v = 4)$

$= (x-4)(x^2 + 4x + 4^2)$

$= (x-4)(x^2 + 4x + 16)$

b) $8w^3 + 27$

$= (2w)^3 + 3^3$ $\quad u = 2w,\ v = 3$

$= (2w+3)((2w)^2 - 3(2w) + 3^2)$

$= (2w+3)(4w^2 - 6w + 9)$

c) $3x^4 + 81x$

$= 3x(x^3 + 27)$

$= 3x(x^3 + 3^3)$

$= \underline{3x(x+3)(x^2 - 3x + 9)}$

d) $2a^3 - 32a$

$= 2a(a^2 - 16)$

$= 2a(a^2 - 4^2)$

$= 2a(a-4)(a+4)$

What about the sum of two squares?

$$x^2 + y^2$$

never factors

idea 1: $(x+y)^2 = x^2 + y^2$?

check: $(x+y)^2 = (x+y)(x+y)$

$= x^2 + xy + xy + y^2$

ex $x^2 + 25$