MATH 1010 ~ Intermediate Algebra

Chapter 3: GRAPHS AND FUNCTIONS

Section 3.4: Equations of Lines

Objectives:

- ✻ Write equations of lines using point-slope form.
- ✻ Write equations of horizontal, vertical, parallel and perpendicular lines.
- ✻ Graph a linear equation without changing the form of the equation.
- ✻ Use linear models to solve application problems.

Point-slope form of an equation of a line: $\boxed{y-y_1 = m(x-x_1)}$

$m = \dfrac{y-y_1}{x-x_1}$

(x_1,y_1) is a point on the line, m is the slope of the line.

x, y variables

Slope-intercept form of an equation is $\boxed{y = mx + b}$
m is the slope and $(0,b)$ is the y-intercept.

General form of an equation of a line: $\boxed{Ax + By + C = 0}$
A, B, and C are integers.

Write the equation of a line with slope m = 3/5 which goes through the point $(\overset{x_1}{-1},\overset{y_1}{2})$ and put it in each of the three forms.

pt-slope form $y-2=\frac{3}{5}(x--1)$

$y-2=\frac{3}{5}x+\frac{3}{5}$
$+2 \qquad +2$

$2+\frac{3}{5}=\frac{10}{5}+\frac{3}{5}=\frac{13}{5}$

slope intercept $y=\frac{3}{5}x+\frac{13}{5}$

$5\left(\frac{-3}{5}x+y-\frac{13}{5}\right)=(0)5$

$-3x+5y-13=0$
General Form

① EXAMPLE

Write the equation in slope-intercept form for the lines containing these pairs of points.

a) $\overset{x_1\ y_1}{(-3,2)}$ and $\overset{x_2\ y_2}{(5,2)}$

$m = \frac{2-2}{5-(-3)} = \frac{0}{8} = 0$

pt slope $y-2 = 0(x - -3)$

$y-2=0$ (General Form)

$y=2$ (slope-int. form)

b) $\overset{x_1\ y_1}{(-3,2)}$ and $\overset{x_2\ y_2}{(-3,5)}$

$m = \frac{5-2}{-3-(-3)} = \frac{3}{0}$ undefined

$\Rightarrow$ vertical line $\boxed{x=-3}$

c) $\overset{x_1\ \ y_1}{(-3/2, -1/2)}$ and $\overset{x_2\ y_2}{(5/8,1/2)}$

$m = \frac{\frac{1}{2}-\frac{-1}{2}}{\frac{5}{8}-\frac{-3}{2}} = \frac{1}{\frac{5}{8}+\frac{3}{2}\left(\frac{4}{4}\right)} = \frac{1}{\frac{5}{8}+\frac{12}{8}} = \frac{1}{17/8} = \frac{8}{17}$

pt slope form

$y - \frac{-1}{2} = \frac{8}{17}\left(x - \frac{-3}{2}\right)$

$y + \frac{1}{2} = \frac{8}{17}x + \frac{\cancel{8}^{4}(3)}{17(\cancel{2})}$

$y + \frac{1}{2} = \frac{8}{17}x + \frac{12}{17}$

$y = \frac{8}{17}x + \frac{12}{17} - \frac{1}{2}$

slope-intercept form $y = \frac{8}{17}x + \frac{7}{34}$

$\frac{12}{17} - \frac{1}{2}$

$= \frac{12}{17}\left(\frac{2}{2}\right) - \frac{1}{2}\left(\frac{17}{17}\right)$

$= \frac{24-17}{34} = \frac{7}{34}$

② EXAMPLE

Write the equation of a line through $\overset{x_1\ y_1}{(3,2)}$ and $\overset{x_2\ y_2}{(5,-4)}$.

State the equation in point-slope form $(y-y_1) = m(x-x_1)$
slope-intercept form $(y = mx+b)$ and
general form $(Ax + By + C = 0)$

$$m = \frac{-4-2}{5-3} = \frac{-6}{2} = -3$$

$y-2 = -3(x-3)$ pt slope form

$$y-2 = -3x+9$$
$$+2 \qquad\qquad +2$$

$y = -3x+11$ slope-intercept form

$$3x+y = 11$$

$3x+y-11=0$ General form

Horizontal and Vertical lines

A horizontal line has an equation in the form: $y = a$

A vertical line has an equation in the form: $x = b$

③ Example

Graph these equations and write the coordinates of three points on each line.

① $x = -2$

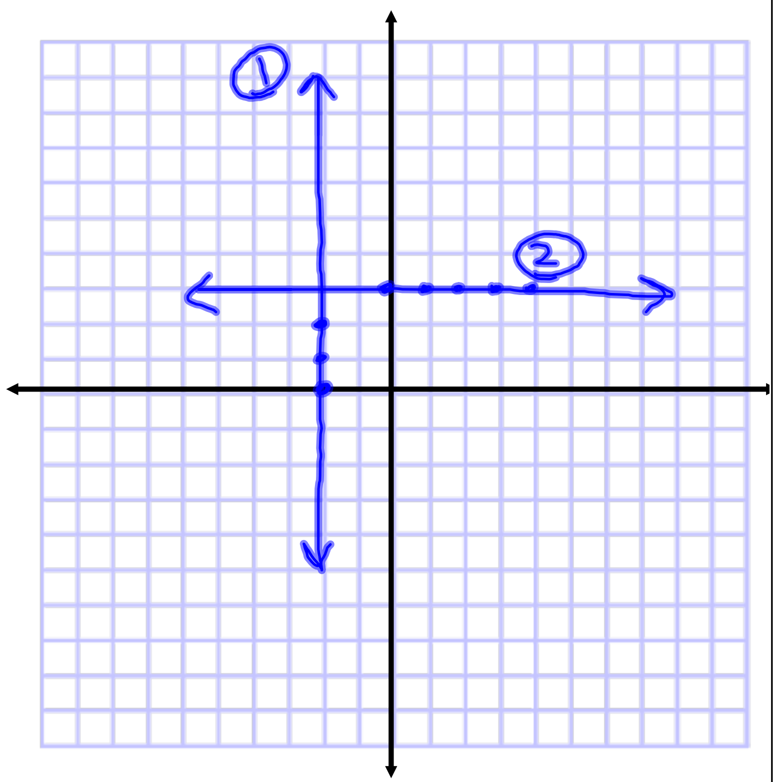
undefined slope (vertical)

x	y
-2	0
-2	1
-2	2
-2	375

② $y = 3$

$m = 0$ (horizontal)

x	y
0	3
1	3
2	3



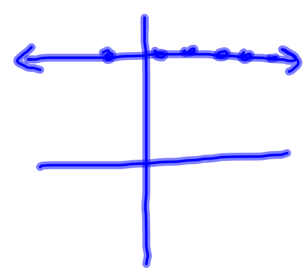
④ EXAMPLE

a) Write an equation of a vertical line through (5,8)

$x = 5$

b) Write an equation of a horizontal line through (-1, 7)

$y = 7$



⑤ EXAMPLE

Find the equation of a line perpendicular to $3x - 4y = 12$ which passes through the point $(-3,6)$

→ slope

$3x - 4y = 12$

$-4y = -3x + 12$

$y = \frac{3}{4}x - 3$

$m = 3/4$

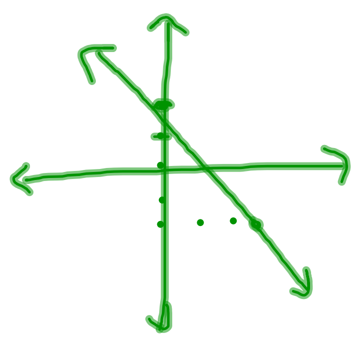
line I want

$m = \frac{-4}{3}$ thru $\overset{x_1}{(-3},\overset{y_1}{6)}$

pt-slope: $y - 6 = \frac{-4}{3}(x - -3)$

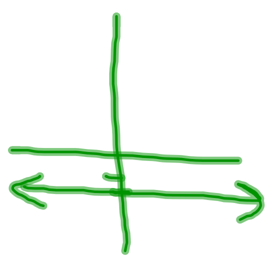
$y - 6 = \frac{-4}{3}x - 4$

$\boxed{y = \frac{-4}{3}x + 2}$



How to sketch a linear equation without changing the form of the equation.

$y = -2$ horiz.



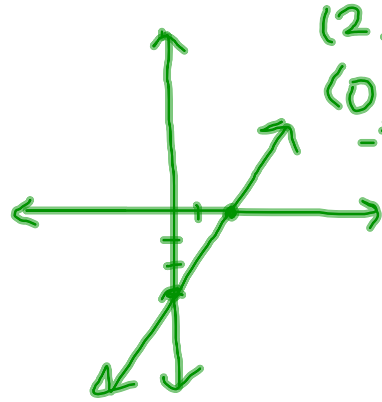
$3x - 2y = 6$

$3x = 6$

$(2, 0)$

$(0, -3)$

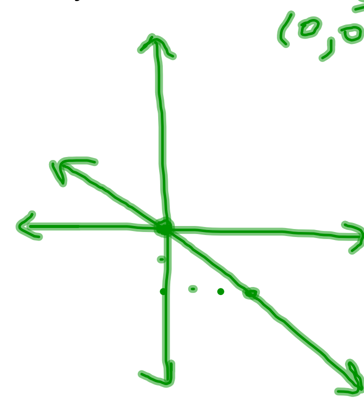
$-2y = 6$



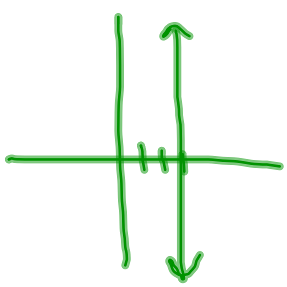
$y = -2/3\ x$

$m = \frac{-2}{3}$

$(0, 0)$



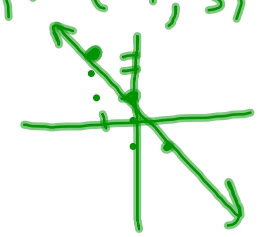
$x = 3$ (vertical)



$y - 3 = -2(x+1)$

(pt slope)

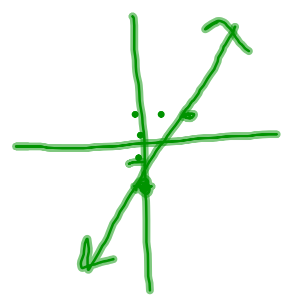
pt $(-1, 3)$ $m = -2$



$y = 3/2\ x - 2$

(slope-int)

$m = 3/2$ $(0, -2)$



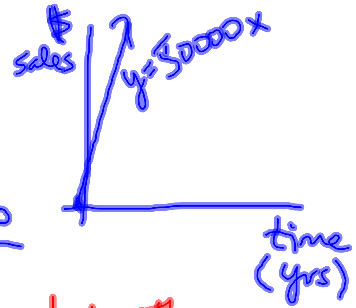
⑥ EXAMPLE

Applications:

a) The total sales for a new sportswear store were $150,000 for the third year and $250,000 for the fifth year. Find a linear model to represent the data. Estimate the total sales for the sixth year.

(t, s)

$\overset{x_1}{(3}, \overset{y_1}{150000)}$ $\overset{x_2}{(5}, \overset{y_2}{250000)}$



$$m = \frac{250000-150000}{5-3} = \frac{100000}{2}$$
$$= 50000$$

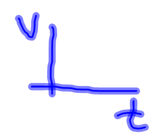
$$y - 150000 = 50000(x-3)$$
$$y - 150000 = 50000x - 150000$$
$$\boxed{y = 50000x}$$

at 6th year
$y = 50000(6)$
$= \$300000$

b) A business purchases a van for $27,500. After 5 years the depreciated value will be $12,000.
Assuming a straight-line depreciation, write an equation of the line giving the value *V* of the van in terms of the time *t* in years.

(t, V)

Use that equation to find the value of the van after 2 years.



$(0, 27500)$ $(5, 12000)$

$$m = \frac{27500-12000}{0-5} = \frac{15500}{-5} = -3100$$

$$V - 27500 = -3100(t-0)$$
$$V - 27500 = -3100t$$
$$V = -3100t + 27500$$

after 2 yrs, $t=2$

$$V = -3100(2) + 27500$$
$$= -6200 + 27500$$
$$= \$21300$$