Utah State Mathematics Contest
Senior Exam
March 17, 2010

1. In the 2010 Vancouver Olympics, placement in curling semi-finals was based on a 10-team round-robin tournament, in which each of 10 teams plays each other team once, and the top four records move on to the semi-finals. A team is considered to be a strong contender for the gold medal if they win 7 of their 9 matchups. In this system, how many different teams can all end the round-robin tournament with a record of 7 or more wins?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

2. The midpoints of the three sides of $\triangle ABC$ are $(5,4)$, $(1,2)$, and $(-1,6)$. Which of the following is one of the actual points A, B, or C?

(a) $(4,9)$ (b) $(3,8)$ (c) $(5,2)$ (d) $(6,3)$ (e) $(0,6)$

3. What is the greatest number of times that the graph of a 10^{th} degree polynomial can intersect with the graph of a 7^{th} degree polynomial?

(a) 3 (b) 7 (c) 10 (d) 17 (e) 70

4. Havermeyer recently inherited a sizeable sum of money. He paid 30% in taxes and invested 20% of what remained into M&M Enterprises. If M&M received $11,900 from Havermeyer, how much did Havermeyer pay in taxes?

(a) $18,200 (b) $19,600 (c) $17,850 (d) $25,500 (e) $3,570

5. Suppose that the Cartesian three-dimensional space is divided up by the planes $x = y$, $x = z$, and $y = z$. How many sections has the 3-space been divided into?

(a) 4 (b) 5 (c) 6 (d) 8 (e) 12

6. Using each of the digits 1 through 9 once, you can create five numbers, each being one or two digits long. If the five numbers are prime, what is the smallest possible sum of these 5 numbers?

(a) 180 (b) 225 (c) 252 (d) 269 (e) 274

7. Let $|\mathbf{U} - \mathbf{10}| = \mathbf{V},$ given that U < 10. What is the value of U – V?

(a) 10 – 2V (b) -10 (c) 10 – 2U (d) 10 (e) $|\mathbf{U} - \mathbf{10}| - \mathbf{U}$

8. In rectangle PQRS, we have P = (4, 16), Q = (24, 26), and S = (7, y) for some integer y < 16. What is the area of rectangle PQRS?

(a) 120 (b) 130 (c) 135 (d) 140 (e) 150

9. For some real number greater than 1, the difference between its multiplicative inverse and its additive inverse is 4. In which interval does the number lie?

(a) [0, 1) (b) [1, 2) (c) [2,3) (d) [3,4) (e) [4,5)

10. A man leaves for work at the same time each morning. If his travel speed averages 30 miles per hour, he will be 18 minutes late. If his travel speed averages 45 miles per hour, he will arrive 8 minutes early. What average travel speed (in miles per hour) will put him at work precisely on time?

(a) 38 (b) 39 (c) 40 (d) 41 (e) 42

11. A regular octagon, ABCDEFGH, has sides of length 8. What is the area of quadrilateral ABDG?

(a) $\mathbf{96 + 64\sqrt{2}}$ (b) $\mathbf{64 + 96\sqrt{2}}$ (c) $\mathbf{32 + 128\sqrt{2}}$

(d) $\mathbf{128 + 32\sqrt{2}}$ (e) $\mathbf{80 + 80\sqrt{2}}$

12. There exists a positive increasing arithmetic sequence A, B, C, D, E such that A, B, E form a geometric sequence. If A = 9, what is D/C?

(a) $\frac{5}{2}$ (b) $\frac{8}{3}$ (c) $\frac{6}{5}$ (d) $\frac{9}{2}$ (e) $\frac{7}{5}$

13. For some integer, n, the quantity (n!) is divisible by the sum (1 + 2 + 3 + ... + n). Which of the following integers is not a possible value of n?

(a) 7 (b) 14 (c) 21 (d) 28 (e) 35

14. Two tour guides are leading seven tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the condition that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

(a) 118 (b) 120 (c) 122 (d) 124 (e) 126

15. How many ordered pairs of positive integers (c, d), where $c > d$, have the property of having their squares differ by 48?

(a) 3 (b) 4 (c) 6 (d) 8 (e) 12

16. Yarrrrrggghhhh! Literary critics complain that 80% of all fictional pirate captains have an eye-patch, 75% have a hook-hand, 67% have a peg-leg, and 90% have a pet parrot. By this accounting, at least what percent of these pirate captains must have an eye-patch, a hook-hand, a peg-leg *and* a parrot?

(a) 7% (b) 12% (c) 18% (d) 26% (e) 36%

17. A trail going up Mount Olympus maintains at a steady grade of 6%. Over the course of this trail, a traveler would rise in elevation by 270 meters. How much longer would the trail be if the trail had a 4% grade?

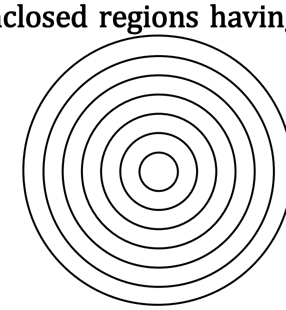
(a) 2250 m (b) 2700 m (c) 3600 m (d) 4500 m (e) 5400 m

18. Suppose that F(x) and G(x) are one-to-one functions. Which of the following is necessarily a one-to-one function?

(a) $(F - G)(x)$ (b) $(F \cdot G)(x)$ (c) $(F / G)(x)$

(d) $(F \circ G)(x)$ (e) $(F + G)(x)$

19. The included picture contains seven concentric circles which have integer radii numbering from 1 to 7. How many non-congruent pairs of enclosed regions having equal area can be found in this picture?



(a) 0 (b) 1 (c) 2

(d) 3 (e) 4

20. A circle passes through the three vertices of an isosceles triangle that has two sides of length 4 and one side of length 2. What is the area of the circle?

(a) $\frac{16\pi}{3}$ (b) $\frac{16\pi}{5}$ (c) $\frac{64\pi}{15}$ (d) $\frac{65\pi}{17}$ (e) $\frac{70\pi}{17}$

21. At one point in US history, it was recommended that roads connect every town to the town nearest to it. Treating each town as a point on a map, and assuming that the distance between any two is distinct (when measured carefully enough), what is the largest number of towns to which any one town may be connected under this system?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

22. The ratio $\frac{20^{2009} + 20^{2011}}{20^{2010} + 20^{2010}}$ is closest to which of the following numbers?

(a) 0.05 (b) 0.1 (c) 1 (d) 10 (e) 20

23. In trapezoid ABCD, line segments AB and CD are both perpendicular to segment AD. As well, $AB + CD = BC$, $AB < CD$, and $AD = 9$. What is $AB \cdot CD$?

(a) 20.25 (b) 22.5 (c) 26.25 (d) 28 (e) 29.5

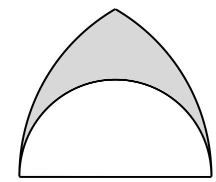
24. Bill randomly selects three different numbers from the set $\{1, 2, 3, 4\}$, while Kerry randomly selects just one number from the set $\{2, 4, 6, 8, 10, 12\}$. What is the probability that Kerry's one number is larger than the sum of Bill's three numbers?

(a) $\frac{3}{8}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{5}{8}$ (e) $\frac{2}{3}$

25. A rare book dealer has packaged his books into six bundles; the bundles contain 15, 16, 18, 19, 20, and 31 books. One of the bundles contains only first-editions, while the other five contain re-prints. One customer buys two bundles of re-prints. Another customer also buys only bundles of re-prints, but buys twice as many books as the first customer. The first-edition bundle contains how many books?

(a) 15 (b) 16 (c) 18 (d) 19 (e) 20

26. The following image has the property that A, B, and C are all equidistant from each other. AC is an arc formed from a circle with center B and radius AB, and BC is an arc formed from a circle with center A and radius AB. The white portion of the picture is a semi-circle with diameter AB. Find the ratio of the area of the shaded region to the area of ABC.



(a) $\frac{4\pi}{6\pi-4\sqrt{3}}$ (b) $\frac{3\pi-3\sqrt{3}}{6\pi-4\sqrt{3}}$ (c) $\frac{\pi}{4\pi-2\sqrt{3}}$

(d) $\frac{2\pi}{12\pi-9\sqrt{3}}$ (e) $\frac{5\pi-6\sqrt{3}}{8\pi-6\sqrt{3}}$

27. The repeating decimal of form of a number is $0.0\overline{378}$... If such a number were converted to a fraction and reduced to lowest form, what would be the denominator?

(a) 185 (b) 333 (c) 370 (d) 1111 (e) 9990

28. The positive integers C, D, C + D, C – D are all prime. The sum of these four primes must be divisible by which of the following?

(a) 2 (b) 3 (c) 5 (d) 7 (e) The sum is prime

29. Evan has the same birthday as his grandson, James. For the last six consecutive years, Evan's age was a multiple of James'. How old will Evan be at the next time that Evan's age will again be a multiple of his grandson's?

(a) 54 (b) 64 (c) 70 (d) 72 (e) 80

30. Let $h(t) = \frac{16t^4+250t}{4t^2-25}$ for $t \neq -\frac{5}{2}$. For $h(t)$ to be continuous, what value must be assigned to $h(t)$ when $t = -\frac{5}{2}$?

(a) $\frac{125}{2}$ (b) $\frac{75}{2}$ (c) $\frac{75}{4}$ (d) $\frac{125}{8}$ (e) $\frac{75}{8}$

31. What is the probability that a randomly selected 10-digit number contains all ten different digits?

(a) $\frac{9\cdot 9!}{10^{10}}$ (b) $\frac{10!-9!}{10^9-1}$ (c) $\frac{9!}{9\cdot 10^9}$ (d) $\frac{10!-9!}{10^{10}-1}$ (e) $\frac{9!}{10^9}$

32. A bag contains two gems, either of which may be a ruby or an emerald (with equal probability). An emerald is added to the bag, the contents are well-shaken, and a random stone is drawn from the bag. If the stone drawn out is an emerald, what is the probability that the bag contains a ruby?

(a) $\frac{5}{7}$ (b) $\frac{7}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) $\frac{5}{8}$

33. Suppose that $30^A = 6$, $30^B = 10$, and $30^C = 15$. What is the mean of A, B, and C?

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{2}{9}$ (e) $\frac{1}{3}$

34. Assuming that A, B are positive integers, which of the following could be the total of the multiplicities of all negative real zeros for the following polynomial?

$$P(x) = x^9 + 4x^8 - 3x^7 - 26x^6 + 3x^5 + 108x^4 + 107x^3 - 46x^2 - Ax - B$$

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

35. One of the two real roots of the quadratic equation, $ax^2 + bx + c = 0$, is three times the other. Which of the following expressions is equal to the discriminant of the quadratic function?

(a) $\frac{4ac-b^2}{2a}$ (b) $\frac{4ac}{3b^2}$ (c) $\frac{3ac}{b^2}$ (d) $\frac{3ac}{2}$ (e) $\frac{4ac}{3}$

36. What portion of all positive integers are not multiples of 2, 3, 4, 5 or 6?

(a) $\frac{7}{30}$ (b) $\frac{4}{15}$ (c) $\frac{3}{10}$ (d) $\frac{1}{3}$ (e) $\frac{11}{30}$

37. Over the interval [0, 4], how many times does the function $f(x) = \frac{1}{4}\cos(4x)$ reach a local minimum?

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

38. Consider the following parametric equations:

$$\mathbf{x = 20\sin t + 5} \qquad\qquad \mathbf{y= -10\cos t - 4}$$

Which of the following best describes the image made by the graph of these equations in the x-y plane?

(a) A circle (b) An ellipse (c) A parabola

(d) A hyperbola (e) A cone

39. The 3rd-degree polynomial function $\mathbf{f(x) = ax^3 + bx^2 + cx + d}$ passes through the points $(-2, 0)$, $(2, 0)$, and $(0, 4)$. What is the value of b?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

40. Once a month, Angus and Maggie hike from their home to a nearby mountain; they proceed up to the top, take the same path back down again, and back home. On the road to and from the mountain, their pace is 4 miles per hour. Going up the mountain, they travel 3 miles per hour, while their speed increases to 6 miles per hour on the trip down. If the whole trip takes 3.5 hours, how many miles is the round-trip?

(a) 14 (b) 15 (c) 16 (d) 18 (e) Not enough information

Utah State Mathematics Contest

Senior Exam Solutions

1. (d) 5

First, each of the five top teams must each beat all of the five bottom teams. As well, each of the five top teams must beat exactly two of the other four top teams. If the teams are labeled A, B, C, D, E, then one such example would be:

A beats B, C B beats C, D C beats D, E D beats A, E E beats A, B

2. (b) (3,8)

If the three points are (A,D), (B,E), (C,F), then the three midpoints are $\left(\frac{A+B}{2}, \frac{D+E}{2}\right)$, $\left(\frac{B+C}{2}, \frac{E+F}{2}\right)$, $\left(\frac{A+C}{2}, \frac{D+F}{2}\right)$. This leads to the following systems of equations:

$$A + B = 10 \qquad D + E = 8$$
$$B + C = 2 \qquad E + F = 4$$
$$A + C = -2 \qquad D + F = 12$$

Solving for the variables gives original points of (3,8), (7,0), (-5,4).

3. (c) 10

An intersection between the original two polynomials is equivalent to a zero in their difference. The difference between a 10th degree polynomial and a 7th degree polynomial is a 10th degree polynomial. A 10th degree polynomial can have at most ten distinct zeros.

4. (d) $25,500

If the inheritance was K, the investment was (0.2)(0.7)K. Taxes were 0.3K, which is equal to $\frac{0.3}{0.14} \cdot 11,900 = 25,500$.

5. (c) 6

The described planes divide 3-space into the following regions:

x>y>z x>z>y y>x>z y>z>x z>x>y z>y>x

6. (b) 225

The four two-digit numbers must end with the digits 1, 3, 7, 9; the one-digit number must be 2 or 5. To minimize the total, the one-digit number should be 5. One possible example that fits these criteria is 5, 23, 41, 67, 89. The minimum total is 225.

7. (a) 10 – 2V

If $U < 10$, then $|U - 10| = 10 - U = V$, so $U = 10 - V$, and $U - V = 10 - 2V$.

8. (e) 150

The slope of $\overline{PQ}$ is $\frac{1}{2}$. Therefore, the slope of $\overline{PS}$ is -2. This forces S to be (7,10). Segments $\overline{PQ}$ & $\overline{PS}$ have lengths $10\sqrt{5}$ & $3\sqrt{5}$, giving an area of 150.

9. (d) [3,4)

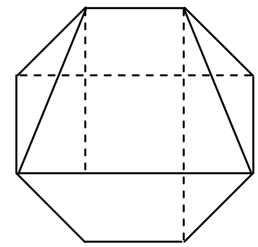
$\left(\frac{1}{x}\right) - (-x) = 4$. Clearing the fraction and converting to standard form leaves $x^2 - 4x + 1 = 0$. The solutions to this equation are $x = 2 \pm \sqrt{3}$. Given that $x > 1$, it must be that $x = 2 + \sqrt{3}$, which lies between 3 and 4.

10. (b) 39

Let T be the amount of time from when the man leaves to when he should be at work. Distance = Speed · Time; $30 \cdot \left(T + \frac{18}{60}\right) = 45 \cdot \left(T - \frac{8}{60}\right)$. Solving gives $T = 1$. Thus, the distance to work is 39 miles, and the necessary speed is 39 mph.

11. (a) $96 + 64\sqrt{2}$

The desired shape is a trapezoid with bases equal to (8) and (8+8$\sqrt{2}$). As well, the height is (8+4$\sqrt{2}$). The area is equal to $\frac{1}{2}(8 + 8 + 8\sqrt{2})(8 + 4\sqrt{2}) = 96 + 64\sqrt{2}$.



12. (e) $\frac{7}{5}$

Let d be the difference between terms in the arithmetic sequence, and let r be the ratio between terms in the geometric sequence. Thus, A + d = Ar, and A + 4d = Ar^2. Rearranging, d = A(r – 1) and 4d = A(r^2 – 1), giving 4A(r – 1) = A(r – 1)(r + 1). Thus, r = 3 and d = 2A. Now, C = 5A and D = 7A; D/C = 7/5.

13. (d) 28

The sum of the first n integers is $\frac{n(n+1)}{2}$. This can only divide (n!) if (n+1) can be factored into at least two smaller factors; (n+1) is factorable for each of the given choices except for 28.

14. (e) 126

Each tourist has two choices, giving 2^7 = 128 groups. However, excluding the two cases of all seven tourists choosing the same guide gives 126 groups.

15. (a) 3

Let c = d + k, where k > 0; then, $c^2 - d^2 = (d + k)^2 - d^2 = k^2 + 2dk = 48$. Then the goal is to factor $k^2 + 2dk - 48$. This is only possible when -48 factors into two even integers with a positive sum; these factors are 8 & -6, 12 & -4, or 24 & -2. Therefore, d is 1, 4, or 11. The appropriate (c,d) pairs are (7,1), (8,4), (13,11).

16. (b) 12%

Let the probabilities of events A and B be P(A) and P(B), respectively. The minimum intersection of two given conditions is the lesser of (P(A) + P(B) – 1) and 0. The minimum intersection of N events is the lesser of *(the sum of the N probabilities minus (N – 1))* and 0. Thus, the minimum intersection of these four conditions is (0.80 + 0.75 + 0.67 + 0.90) – 3 = 0.12.

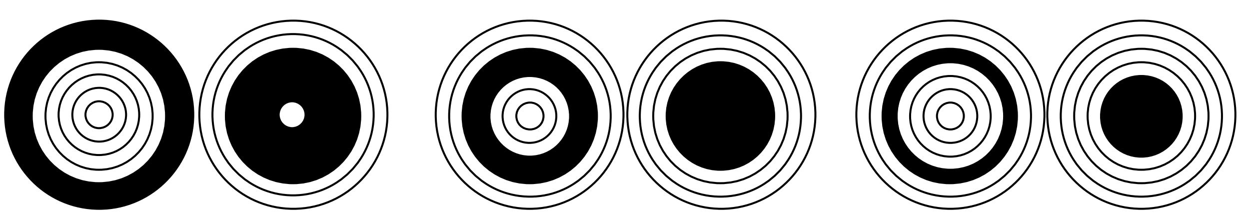
17. (a) 2250 m

The horizontal distance traveled at a 6% grade would be $\frac{270}{.06} = 4500$. The horizontal distance traveled at a 4% grade would be $\frac{270}{.04} = 6750$. At such low grade, the difference between the horizontal distance and the trail distance is negligible. Thus, the difference in trail length is almost exactly 2250 meters.

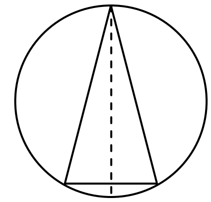
18. (d) $(F \circ G)(x)$

Addition, subtraction, multiplication, and division of the right examples can easily produce a result of a constant function, which is not one-to-one. However, the composition of one-to-one functions must necessarily be one-to-one.

19. (d) 3



20. (c) $\frac{64\pi}{15}$



The height of the triangle is $\sqrt{15}$. Let x be the length of the dotted segment from the bottom of the triangle to the edge of the circle. The Chord Theorem states that $\sqrt{15} \cdot x = 1 \cdot 1$, giving $x = \frac{1}{\sqrt{15}}$.

The area of the circle is $\pi \cdot \left(\frac{1}{2}\left(\sqrt{15} + \frac{1}{\sqrt{15}}\right)\right)^2 = \frac{64\pi}{15}$.

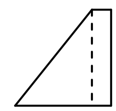
21. (c) 5

Imagine that roads between towns are completely straight. If the angle formed by the roads between a central town and two smaller towns is less than 60°, then the distance between the two smaller towns is less than the distance from the further small town to the central town. If a central town is attached by road to six or more towns, then two or more of the attached towns will be within 60° of each other.

22. (d) 10

The fraction can be factored into $\frac{20^{2009}(1+400)}{20^{2009}(20+20)} = \frac{401}{40} \approx \mathbf{10}$.

23. (a) 20.25



Call the top base (x), and the bottom base (x+y). The diagonal line is (2x+y). The height of the trapezoid is 9. In the right triangle, $81 = (2x+y)^2 - y^2 = (4x^2 + 4xy) = 4x(x+y) = 4\overline{AB} \cdot \overline{CD}$. Thus, $\overline{AB} \cdot \overline{CD} = \frac{81}{4}$.

24. (c) $\frac{5}{12}$

Bill's total must be 6, 7, 8 or 9. Kerry's probability of beating Bill's number is

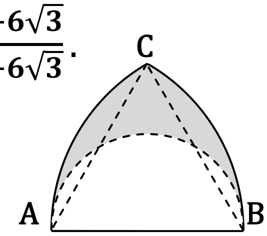
$$\frac{0}{24} + \frac{0}{24} + \frac{0}{24} + \frac{2}{24} + \frac{4}{24} + \frac{4}{24} = \frac{10}{24} = \frac{5}{12}$$

25. (e) 20

One buyer buys two bundles, and the other buys twice as many books. With the given numbers, the second buyer must buy three bundles. The number of books in the five purchased bundles must be a multiple of three; as number of books in the six bundles are 0, 1, 0, 1, 2, & 1 mod 3, this is only possible if the un-bought bundle contains 20 (2 mod 3) books. Thus, the un-bought first-edition bundle contains 20 books.

26. (e) $\frac{5\pi - 6\sqrt{3}}{8\pi - 6\sqrt{3}}$

Call the radius of the semi-circle R. The area of ABC is the sum of the areas of two 60° arcs from circles of radius 2R minus their overlap of an equilateral triangle with side-length 2R. The area of each arc is $\frac{4\pi R^2}{6}$, and the area of the triangle is $\sqrt{3}R^2$. Hence, the area of ABC is $\frac{(8\pi - 6\sqrt{3})}{6}R^2$. The area of the shaded region is $\frac{(8\pi - 6\sqrt{3})}{6}R^2 - \frac{\pi}{2}R^2 = \frac{(5\pi - 6\sqrt{3})}{6}R^2$. The ratio of these two areas, after cancelling, is $\frac{5\pi - 6\sqrt{3}}{8\pi - 6\sqrt{3}}$.



27. (a) 185

Converting a decimal to a fraction depends on the number of digits until the repeating starts and the number of digits it takes to repeat. The given decimal is $\frac{378}{9990} = \frac{7}{185}$.

28. (e) The sum is prime

(C – D), (C), (C+D) can only all be prime if D is even. Since D must also be prime, D is 2. As well, the only string of three consecutive odd numbers all being prime is 3, 5, 7 (every third odd must be a multiple of 3). So, the sum is 17.

29. (c) 70

To meet the given conditions, Evan's age directly before the string of consecutive years must have been a multiple of 1, 2, 3, 4, 5, 6. The only human ages where this would be possible would be 60 and 120. Barring the extreme example (only two people in recorded history have been verified as reaching age 120), Evan is 60 years older than James. When Evan is 70, James will be 10.

30. (b) $\frac{75}{2}$

When $t \neq -\frac{5}{2}$, $h(t) = \frac{16t^4+250t}{4t^2-25}$ simplifies to $\frac{2t(4t^2-10t+25)}{2t-5}$. Evaluating this fraction at $t = -\frac{5}{2}$ gives a value of $\frac{75}{2}$.

31. (e) $\frac{9!}{10^9}$

If you don't allow the first digit to be 0, then the probability is $\frac{9 \cdot 9!}{9 \cdot 10^9}$. If you do let the first digit be 0, the probability is $\frac{10!}{10^{10}}$. Either way, the fraction simplifies to (e).

32. (e) $\frac{5}{8}$

At the time of drawing, there are three possibilities. The probability of the bag containing three emeralds and of drawing an emerald is $\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{3}\right) = \frac{3}{12}$; the probability of the bag containing two emeralds and of drawing an emerald is $2 \cdot \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{3}\right) = \frac{4}{12}$; the probability of the bag containing one emerald and of drawing an emerald is $\left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3}\right) = \frac{1}{12}$. Therefore, the probability that the bag still contains a ruby is $\frac{\frac{4}{12}+\frac{1}{12}}{\frac{3}{12}+\frac{4}{12}+\frac{1}{12}} = \frac{5}{8}$.

33. (c) $\frac{2}{3}$

Multiply $30^A \cdot 30^B \cdot 30^C = 30^{A+B+C} = 900$. Therefore, (A+B+C)=2, and the mean of A, B, an C is 2/3.

34. (b) 6

P(-x) contains six sign changes. By Descartes' Rule of Signs, the number of negative roots (allowing for multiplicity) must be 6 or a non-negative even number less than 6. Since no other such numbers are available, the only valid choice is (b).

35. (e) $\frac{\mathbf{4ac}}{\mathbf{3}}$

Let the roots be d and 3d. Then, $\mathbf{ax^2+bx+c=a(x-d)(x-3d)=0}$, $\mathbf{b\ =\ -4ad}$, and $\mathbf{c=3ad^2}$. The discriminant, $\mathbf{b^2-4ac}$, is equal to $\mathbf{16a^2d^2-12a^2d^2=4a^2d^2=4a\cdot\frac{c}{3}}$, or option (e).

36. (b) $\frac{\mathbf{4}}{\mathbf{15}}$

The multiples of 4 and 6 would all be subsets of multiples of 2, so they are irrelevant to the question. The portion of positive integers that are multiples of 2, 3, and 5 can be calculated by combining relative portions and removing overlapping portions: $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}-\frac{1}{2}\cdot\frac{1}{3}-\frac{1}{2}\cdot\frac{1}{5}-\frac{1}{3}\cdot\frac{1}{5}+\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{5}=\frac{15}{30}+\frac{10}{30}+\frac{6}{30}-\frac{5}{30}-\frac{3}{30}-\frac{2}{30}+\frac{1}{30}=\frac{22}{30}$.

The remaining portion $\left(\frac{8}{30}\right)$ of integers contains no multiples of 2, 3, 4, 5 or 6.

37. (c) 3

The function cos(x) reaches local minimums at x=π, 3π, 5π, etc. The function cos(4x) reaches local minimums four times as often, or at $x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4}$, etc. Specifically, the three x-values $\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4}$ all fall in the interval [0, 4], which can be verified by assuming that π≈3.14.

38. (b) An ellipse

Solving for the trigonometric portions of the equations, $\mathbf{\sin t}=\frac{x-5}{20}$ and $\mathbf{\cos t}=-\frac{y+4}{10}$, giving that $\mathbf{\sin^2 t+\cos^2 t}=\left(\frac{x-5}{20}\right)^2+\left(-\frac{y+4}{10}\right)^2=\mathbf{1}$, which is the equation for an ellipse in standard form.

39. (b) -1

$\mathbf{f(2)+f(-2)=\ 8a+4b+2c+d-8a+4b-2c+d=8b+2d=0.}$

$\mathbf{f(0)=d=4}$. Substituting, $\mathbf{8b+8=0}$, or $\mathbf{b=-1}$.

40. (a) 14

Let the x be the distance up the mountain. Total time spent traveling on the mountain is $\frac{x}{3}+\frac{x}{6}=\frac{3x}{6}=\frac{x}{2}$. Total distance traveled on the mountain is 2x. Therefore, the average speed on the mountain is $\frac{2x}{\frac{x}{2}}=\mathbf{4}$. As the average speed on the mountain and the average speed on the road are both 4 miles per hour, then the average speed for the whole trip is 4 mph. It follows that the total distance traveled is $3.5\cdot4=14$ miles.