$\mathbb{Z}/p$-acyclic resolutions in the "strongly countable" $\mathbb{Z}/p$-dimensional case

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Joint work with Leonard Rubin, University of Oklahoma

EXTENDED VERSION of the 15 min talk

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Definitions

Before stating the theorem that produced our title:

$\mathbb{Z}/p$-acyclic resolutions in the "strongly countable" $\mathbb{Z}/p$-dimensional case

we will need to define what is:

- a resolution
- dim and $\dim_G$ ($\dim_{\mathbb{Z}/p}$)
- a cell-like map
- a G-acyclic map ($\mathbb{Z}/p$-acyclic map)
- strong countability – we are not using this notion in its original form– these words refer to the infinite sequence of closed spaces $X_1 \subset X_2 \subset \ldots$ with finite $\dim_{\mathbb{Z}/p}$ in the statement of our theorem

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The resolution we obtain will be between a domain Z of finite dim, and a range X of finite $\dim_G$, with cell-like or G-acyclic fibers.

Both domain and range will be compact metrizable spaces.
All groups we refer to will be abelian.

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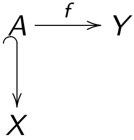

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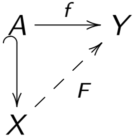

Standard notation: $Y \in \mathrm{AE}(X)$.
Also used: $\text{e-dim}\, X \leq Y$.
We will use: $X \tau Y$.

Characterization of dim and $\dim_G$ by extension of maps

Theorem

For any nonempty paracompact Hausdorff space X *and* $n \in \mathbb{Z}_{\geq 0}$,

- $\dim X \leq n \;\Leftrightarrow\; X \tau S^n$,
- *for any abelian group* G, $\dim_G X \leq n \;\Leftrightarrow\; X \;\tau\; K(G,n)$.

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$K(G, n)$ = an Eilenberg-MacLane complex of type (G, n)
= a connected CW-complex having the property

$$\pi_i(K(G, n)) \cong \begin{cases} G & \text{if } i = n \\ 0 & \text{if } i \neq n. \end{cases}$$

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- for a compact metrizable space X,

$$\dim_G X \leq \dim_{\mathbb{Z}} X \leq \dim X$$

- if X is a compact metrizable space with $\dim X < \infty$, then $\dim_{\mathbb{Z}} X = \dim X$ (Thm by Aleksandrov)
- there are compact metrizable spaces with infinite dim and finite $\dim_{\mathbb{Z}}$ (Eg by Dranishnikov, Dydak-Walsh)

Definition

A map $\pi : Z \to X$ between compact spaces is called cell-like if each of its fibers $\pi^{-1}(x)$ is a cell-like set, i.e., for any CW-complex K and any $x \in X$, every map $f : \pi^{-1}(x) \to K$ is nullhomotopic. Or, equivalently, every fiber $\pi^{-1}(x)$ has the shape of a point.

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Clearly, $\pi : Z \to X$ is cell-like $\Rightarrow$ π is G-acyclic.

Theorem (R. Edwards - J. Walsh, 1981)

For every compact metrizable space X with $\dim_{\mathbb{Z}} X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \to X$ such that π is cell-like, and $\dim Z \leq n$.

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Z	$\dim Z \leq n$	$\dim Z \leq n$
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Resolution Theorems

Edwards-Walsh, Dranishnikov

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Theorem (A. Dranishnikov, 1988)

For every compact metrizable space X with $\dim_{\mathbb{Z}/p} X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \to X$ such that π is $\mathbb{Z}/p$-acyclic, and $\dim Z \leq n$.

Theorem (M. Levin, 2005)

Let $n \in \mathbb{N}_{\geq 2}$. Then for every compact metrizable space X with $\dim_{\mathbb{Q}} X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \longrightarrow X$ such that π is $\mathbb{Q}$-acyclic, and $\dim Z \leq n$.

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Resolution Theorems

Levin Resolution Theorem for $\mathbb{Q}$

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$$Z \xrightarrow{\ \pi\ } X \qquad \dim Z \leq n \xrightarrow{\text{cell-like}} \dim_{\mathbb{Z}} X \leq n \qquad \dim Z \leq n \xrightarrow{\mathbb{Z}/p\text{-acyclic}} \dim_{\mathbb{Z}/p} X \leq n \qquad \dim Z \leq n \xrightarrow{\mathbb{Q}\text{-acyclic}} \dim_{\mathbb{Q}} X \leq n$$

This does not work for any abelian group G:
if $G = \mathbb{Z}/p^{\infty} = \{\frac{m}{n} \in \mathbb{Q}/\mathbb{Z} \ : \ n = p^k \text{ for some } k \geq 0\}$
(quasi-cyclic p-group), then $\dim Z \not\leq n$, but $\dim Z \leq n+1$.

Theorem (M. Levin, 2003)

Let G be an abelian group, $n \in \mathbb{N}_{\geq 2}$. Then for every compact metrizable space X with $\dim_G X \leq n$, there exists a compact metrizable space Z and a surjective map $\pi : Z \to X$ such that:
(a) π *is G-acyclic,*
(b) $\dim Z \leq n + 1$, *and*
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Possible generalization

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$$Z$$

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$$\begin{array}{ccc} Z' & \hookrightarrow & Z \\ {\scriptstyle \pi|}\downarrow & & \downarrow{\scriptstyle \pi} \\ X' & \hookrightarrow & X \end{array}$$

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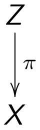

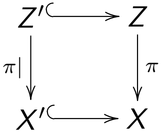

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Possible generalization

Ageev-Jiménez-Rubin Theorem for $\mathbb{Z}$

$\dim Z_1 \leq 1 \quad \dim Z_2 \leq 2 \quad \dim Z_k \leq k$

$$Z_1 \quad Z_2 \quad \cdots \quad Z_k \quad \cdots \quad Z$$

$$X_1 \hookrightarrow X_2 \hookrightarrow \cdots \hookrightarrow X_k \hookrightarrow \cdots \hookrightarrow X$$

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Theorem (S. Ageev, R. Jiménez and L. Rubin, 2004)

Let X be a nonempty compact metrizable space and let $X_1 \subset X_2 \subset \ldots$ be a sequence of nonempty closed subspaces such that $\forall k \in \mathbb{N}$, $\dim_{\mathbb{Z}} X_k \leq k < \infty$. Then there exists a compact metrizable space Z, having closed subspaces $Z_1 \subset Z_2 \subset \ldots$, and a (surjective) cell-like map $\pi : Z \to X$, s.t. $\forall k \in \mathbb{N}$,

(a) $\dim Z_k \leq k$,

(b) $\pi(Z_k) = X_k$, *and*

(c) $\pi|_{Z_k} : Z_k \to X_k$ *is a cell-like map.*

Possible generalization
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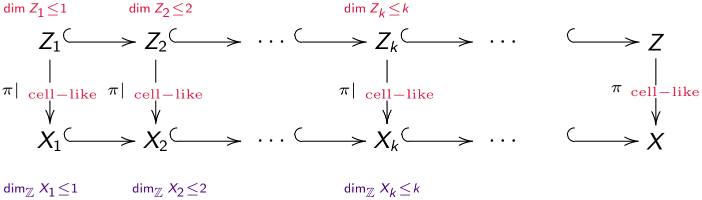

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Possible generalization

Rubin-T. Theorem for $\mathbb{Z}/p$

$$\begin{array}{ccccccccc} \dim Z_1 \le \ell_1 & & \dim Z_2 \le \ell_2 & & & & \dim Z_k \le \ell_k & & \\ Z_1 & & Z_2 & & \cdots & & Z_k & \cdots & Z \end{array}$$

$$X_1 \hookrightarrow X_2 \hookrightarrow \cdots \hookrightarrow X_k \hookrightarrow \cdots \hookrightarrow X$$

$\dim_{\mathbb{Z}/p} X_1 \le \ell_1$ $\dim_{\mathbb{Z}/p} X_2 \le \ell_2$ $\dim_{\mathbb{Z}/p} X_k \le \ell_k$

Theorem (L. Rubin and V. T., 2010)

Let X be a nonempty compact metrizable space, let $\ell_1 \le \ell_2 \le \dots$ be a sequence of natural numbers, and let $X_1 \subset X_2 \subset \dots$ be a sequence of nonempty closed subspaces of X such that $\forall k \in \mathbb{N}$, $\dim_{\mathbb{Z}/p} X_k \le \ell_k < \infty$. Then there exists a compact metrizable space Z, having closed subspaces $Z_1 \subset Z_2 \subset \dots$, and a (surjective) cell-like map $\pi : Z \to X$, such that for each k in $\mathbb{N}$,

(a) $\dim Z_k \le \ell_k$,

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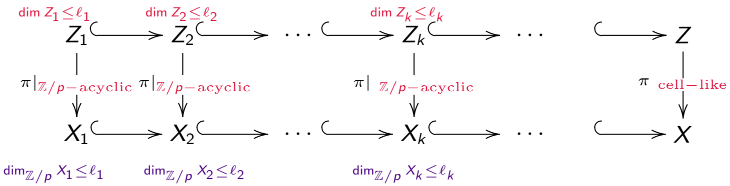

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Possible generalization

Conjecture for any abelian group G

$\dim Z_1 \leq \ell_1 + 1$ $\quad$ $\dim Z_2 \leq \ell_2 + 1$ $\quad$ $\dim Z_k \leq \ell_k + 1$

$\dim_G Z_1 \leq \ell_1$ $\quad$ $\dim_G Z_2 \leq \ell_2$ $\quad$ $\dim_G Z_k \leq \ell_k$

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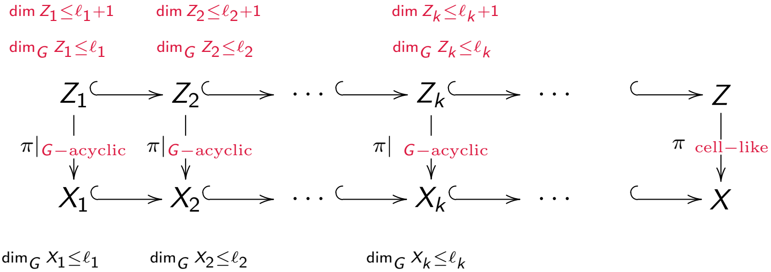
$$Z_1 \qquad Z_2 \qquad \cdots \qquad Z_k \qquad \cdots \qquad Z$$

$$X_1 \hookrightarrow X_2 \hookrightarrow \cdots \hookrightarrow X_k \hookrightarrow \cdots \hookrightarrow X$$

$\dim_G X_1 \leq \ell_1$ $\quad$ $\dim_G X_2 \leq \ell_2$ $\quad$ $\dim_G X_k \leq \ell_k$

where $\ell_1 \leq \ell_2 \leq \cdots \leq \ell_k \leq \ldots$ is a sequence of numbers in $\mathbb{N}_{\geq 2}$.

This would be a generalization of Levin's theorem for any abelian group G.



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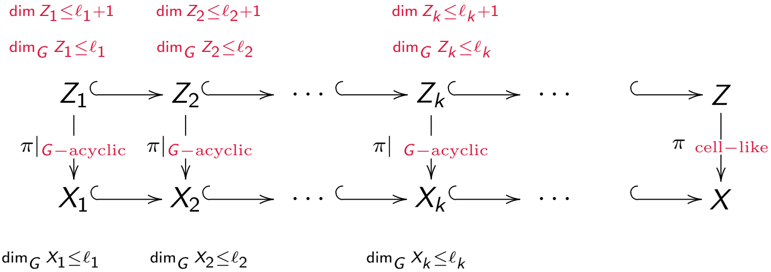

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Techniques used in proofs of resolution theorems

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Theorem (H. Freudenthal, 1937)

Every compact metrizable space can be represented as the inverse limit of an inverse sequence of compact polyhedra, with surjective and simplicial bonding maps.

Techniques used in proofs of resolution theorems

General idea for proving resolution theorems:
Let X be a compact metrizable space with the required property (eg. $\dim_G X \leq n$).

Theorem (H. Freudenthal, 1937)

Every compact metrizable space can be represented as the inverse limit of an inverse sequence of compact polyhedra, with surjective and simplicial bonding maps.

$$P_1 \xleftarrow[f_1^2]{} P_2 \xleftarrow[f_2^3]{} \cdots \xleftarrow[f_{i-1}^i]{} P_i \xleftarrow[f_i^{i+1}]{} P_{i+1} \longleftarrow \cdots \qquad X$$

(1) Choose an inverse sequence (P_i, f_i^{i+1}) of compact polyhedra, with simplicial, surjective bonding maps, whose limit is X.

Techniques used in proofs of resolution theorems

$$
\begin{array}{ccccccccccc}
M_1 & & M_2 & & \cdots & & M_i & & M_{i+1} & & & & Z \\
\\
P_1 & \xleftarrow[f_1^2]{} & P_2 & \xleftarrow[f_2^3]{} & \cdots & \xleftarrow[f_{i-1}^i]{} & P_i & \xleftarrow[f_i^{i+1}]{} & P_{i+1} & \xleftarrow{} & \cdots & & X
\end{array}
$$

(2) Use this sequence as a foundation to build another inverse sequence (M_i, g_i^{i+1}) and an almost commutative ladder of maps, so that $\lim(M_i, g_i^{i+1}) = Z$ and the map $\pi : Z \to X$ with desired properties can be produced.

Techniques used in proofs of resolution theorems

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\begin{array}{ccccccccccc}
M_1 & \xleftarrow{g_1^2} & M_2 & \xleftarrow{g_2^3} & \cdots & \xleftarrow{g_{i-1}^i} & M_i & \xleftarrow{g_i^{i+1}} & M_{i+1} & \longleftarrow & \cdots & & Z \\
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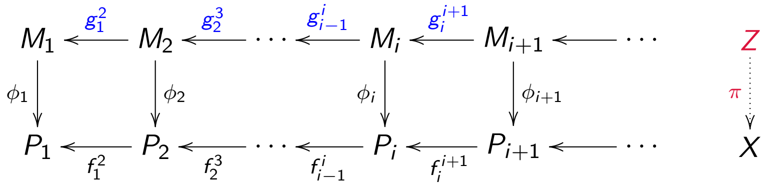

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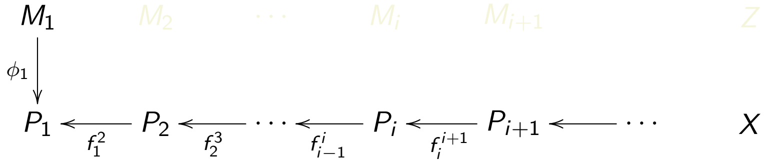

The bottom inverse sequence is pre-chosen, while the top inverse sequence and the ladder of maps are built gradually.

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\begin{array}{ccccccccccc}
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\downarrow{\scriptstyle \phi_1} & & \downarrow{\scriptstyle \phi_2} & & & & & & & & \\
P_1 & \xleftarrow[f_1^2]{} & P_2 & \xleftarrow[f_2^3]{} & \cdots & \xleftarrow[f_{i-1}^i]{} & P_i & \xleftarrow[f_i^{i+1}]{} & P_{i+1} & \longleftarrow \cdots & X
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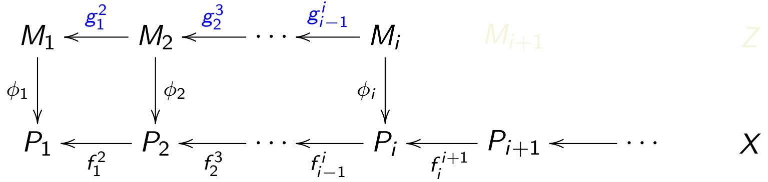

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About the proof of Rubin-T. Theorem

Part of the construction done in Hilbert cube $I^{\aleph_0} = Q = I^m \times Q_m$ with metric $\rho(x, y) = \sum_{i=1}^{\infty} \frac{|x_i - y_i|}{2^i}$.

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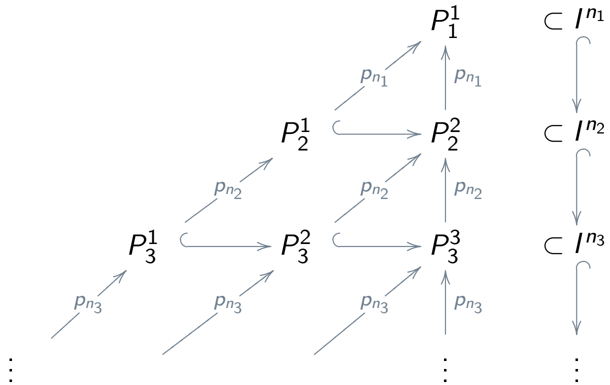
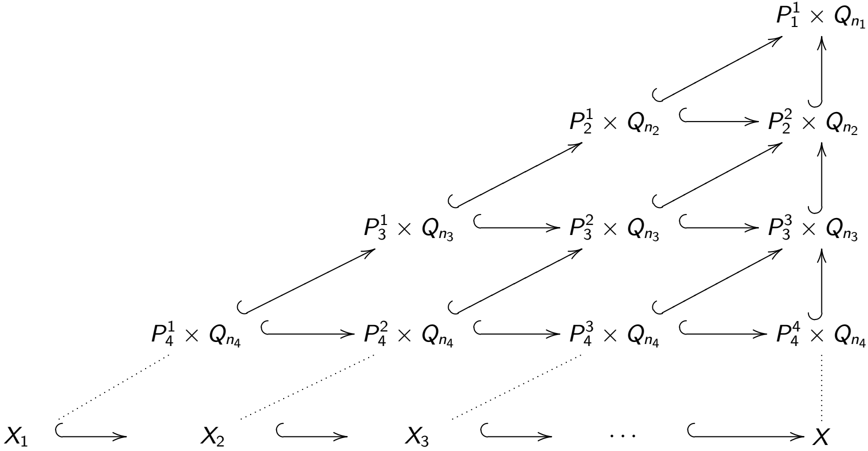


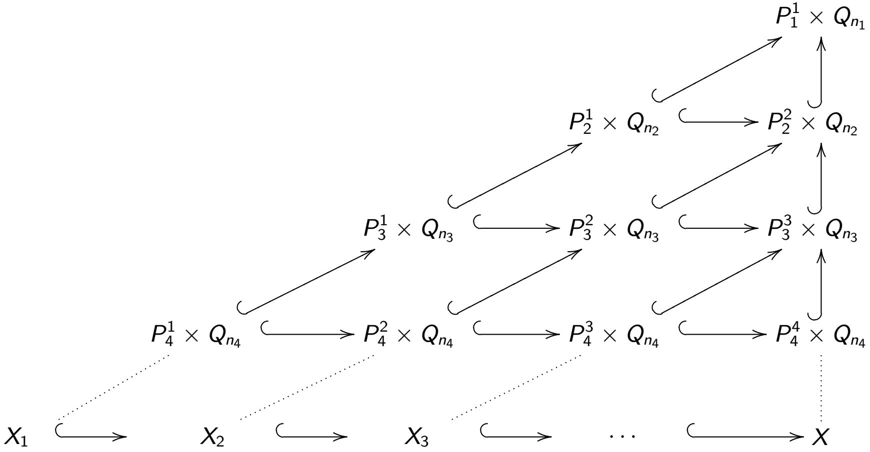
About the proof of Rubin-T. Theorem

... so that $X = \bigcap_{i=1}^{\infty} P_i^i \times Q_{n_i}$, and $X_k = \bigcap_{i=k}^{\infty} P_i^k \times Q_{n_i}$.



About the proof of Rubin-T. Theorem

Instead of the bottom inverse sequence we now have:



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While choosing polyhedra, we simultaneously build simplicial maps $g_i^{i+1} : P_{i+1}^{i+1} \longrightarrow P_i^i$:

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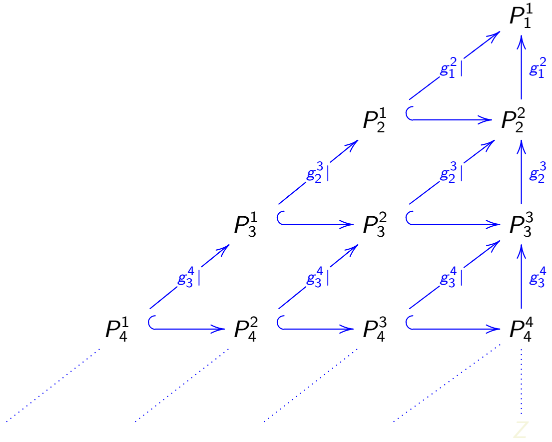

About the proof of Rubin-T. Theorem

Our goal is: $Z := \lim(P_i^i, g_i^{i+1})$

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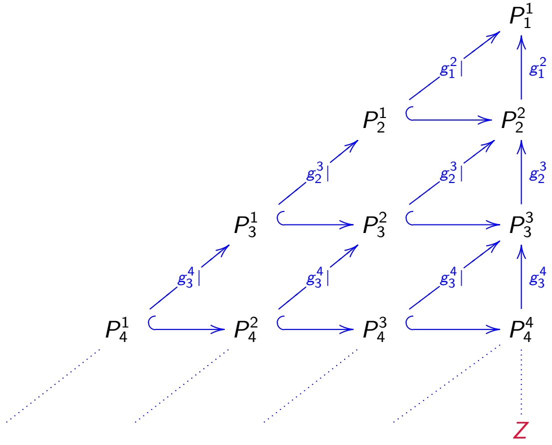

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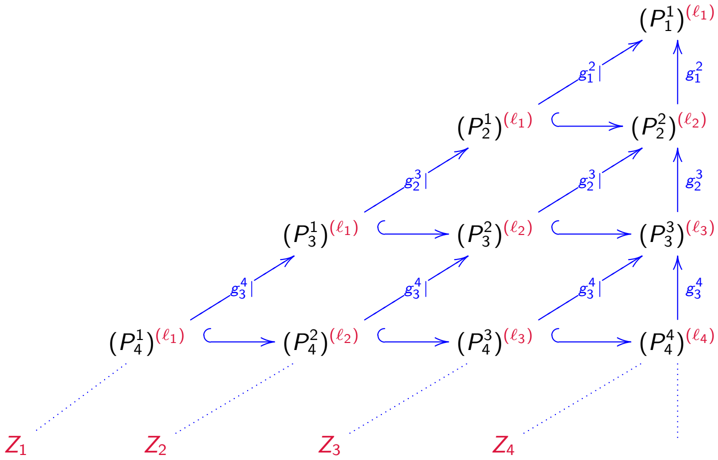

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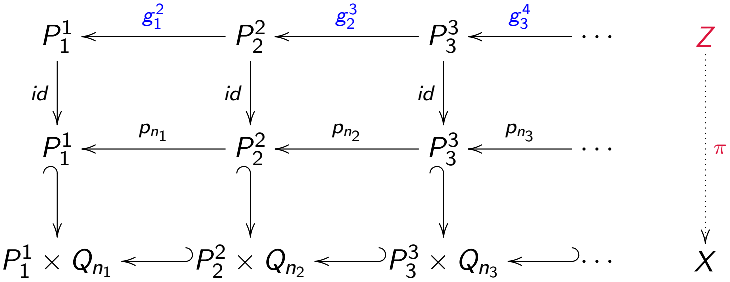

About the proof of Rubin-T. Theorem

We choose both the polyhedra P_i^i and the maps $g_i^{i+1} : P_{i+1}^{i+1} \to P_i^i$ as we go (the bottom sequence is not pre-chosen).

$$
\begin{array}{ccccccccc}
P_1^1 & \xleftarrow{g_1^2} & P_2^2 & \xleftarrow{g_2^3} & P_3^3 & \xleftarrow{g_3^4} & \cdots & & Z \\
\downarrow{\scriptstyle id} & & \downarrow{\scriptstyle id} & & \downarrow{\scriptstyle id} & & & & \vdots \\
P_1^1 & \xleftarrow{p_{n_1}} & P_2^2 & \xleftarrow{p_{n_2}} & P_3^3 & \xleftarrow{p_{n_3}} & \cdots & & \vdots\,\pi \\
\downarrow & & \downarrow & & \downarrow & & & & \downarrow \\
P_1^1 \times Q_{n_1} & \hookleftarrow & P_2^2 \times Q_{n_2} & \hookleftarrow & P_3^3 \times Q_{n_3} & \hookleftarrow & \cdots & & X
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$$\begin{array}{c} \mathrm{EW}(L, G, n) \\ \big\downarrow{\scriptstyle \omega} \\ |L| \end{array}$$

For G an abelian group, $n \in \mathbb{N}$ and L a simplicial complex, an Edwards-Walsh resolution of L in dimension n is a pair $(\mathrm{EW}(L, G, n), \omega)$ consisting of a CW-complex $\mathrm{EW}(L, G, n)$ and a combinatorial map $\omega : \mathrm{EW}(L, G, n) \to |L|$ (that is, for each subcomplex L' of L, $\omega^{-1}(|L'|)$ is a subcomplex of $\mathrm{EW}(L, G, n)$) such that:

$$\begin{array}{ccccc} \mathrm{EW}(L,G,n) & & \omega^{-1}(|L'|) & & \\ \big\downarrow{\scriptstyle \omega} & & & & \\ |L| & & |L'| & & K(G,n) \end{array}$$

(i) $\omega^{-1}(|L^{(n)}|) = |L^{(n)}|$ and $\omega|_{|L^{(n)}|}$ is the identity map of $|L^{(n)}|$ onto itself,

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(ii) for every simplex σ of L with $\dim \sigma > n$, the preimage $\omega^{-1}(\sigma)$ is an Eilenberg-MacLane complex of type $(\bigoplus G, n)$, where the sum here is finite, and

About the proof of Rubin-T. Theorem

Edwards-Walsh complexes

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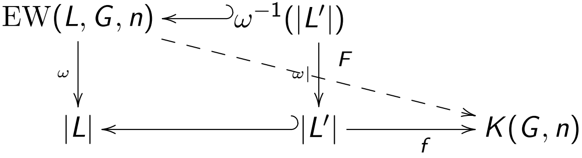
(iii) for every subcomplex L' of L and every map $f : |L'| \to K(G, n)$, the composition $f \circ \omega|_{\omega^{-1}(|L'|)} : \omega^{-1}(|L'|) \to K(G, n)$ extends to a map $F : \mathrm{EW}(L, G, n) \to K(G, n)$.



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Lemma

For the groups $\mathbb{Z}$ and $\mathbb{Z}/p$, for any $n \in \mathbb{N}$ and for any simplicial complex L, there is an Edwards–Walsh resolution $\omega : \mathrm{EW}(L, G, n) \to |L|$ with the additional property for $n > 1$:

1. *the $(n+1)$-skeleton of $\mathrm{EW}(L, \mathbb{Z}, n)$ is equal to $L^{(n)}$;*
2. *the $(n+1)$-skeleton of $\mathrm{EW}(L, \mathbb{Z}/p, n)$ is obtained from $L^{(n)}$ by attaching $(n+1)$-cells by a map of degree p to the boundary $\partial\sigma$, for every $(n+1)$-dimensional simplex σ.*

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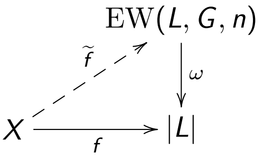
Describe how to build an $\mathrm{EW}(L, \mathbb{Z}/p, n)$.

Edwards-Walsh complexes (resolutions) are useful because

Lemma

Let X be a compact metrizable space with $\dim_G X \leq n$, and let L be a finite simplicial complex. Then for every Edwards-Walsh resolution $\omega : \mathrm{EW}(L, G, n) \to |L|$, and for every map $f : X \to |L|$, there exists an approximate lift $\widetilde{f} : X \to \mathrm{EW}(L, G, n)$ of f.



$\dim_G X \leq n \Leftrightarrow X \tau K(G, n)$

$\widetilde{f}$ is an approximate lift of f w.r. to ω if $\forall x \in X,\ f(x) \in \Delta \Rightarrow \omega \circ \widetilde{f}(x) \in \Delta$.

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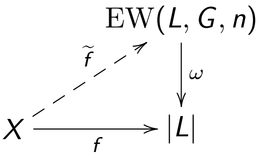

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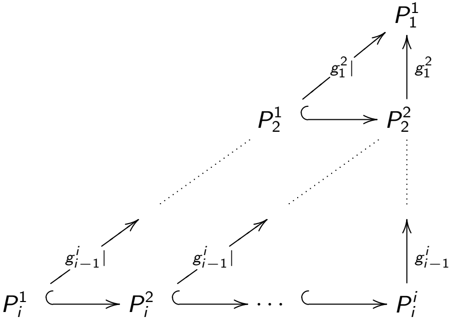
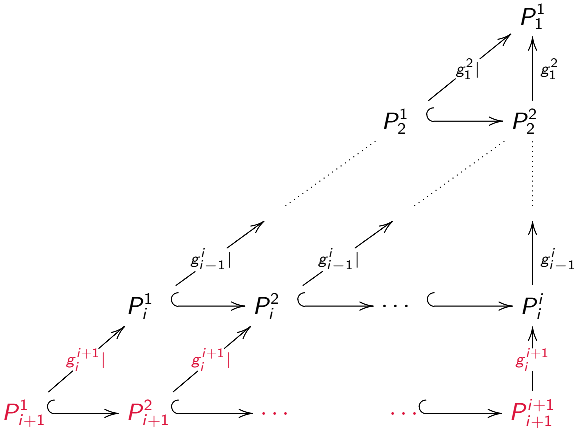


We would like to build:



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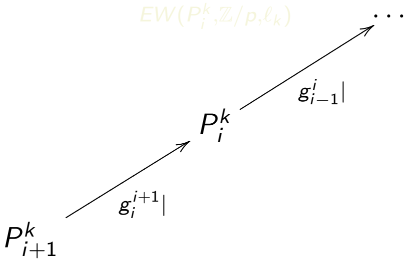

within each of our diagonals, we need to have that, for infinitely many indexes i, $g_i^{i+1}|$ factors up to homotopy through an Edwards-Walsh complex:

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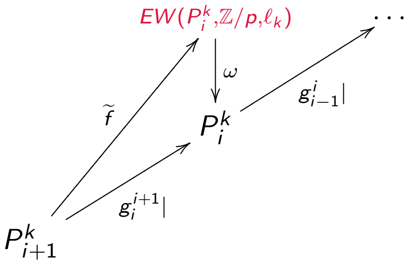

within each of our diagonals, we need to have that, for infinitely many indexes i, $g_i^{i+1}|$ factors up to homotopy through an Edwards-Walsh complex: $g_i^{i+1}| \simeq \omega \circ \widetilde{f}$.

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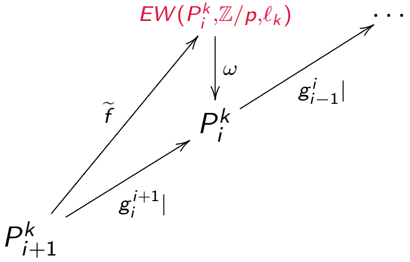

So we will have to choose a "book-keeping" function $\nu : \mathbb{N} \to \mathbb{N}$ to tell us on which diagonal to focus next.

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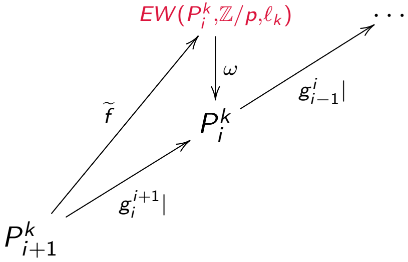

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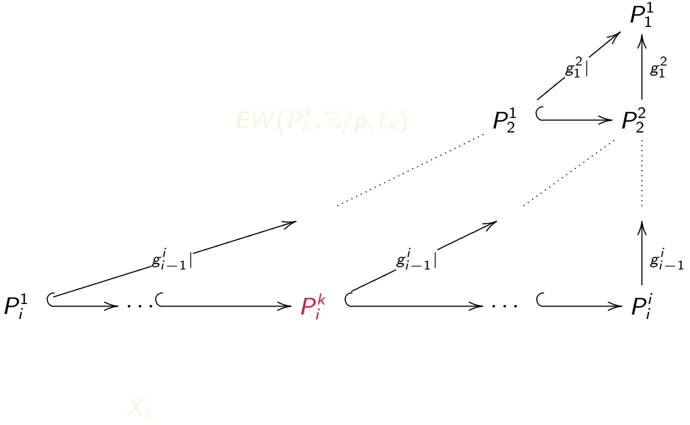
$\nu(i) \leq i$, $\nu^{-1}(k)$ is infinite.

About the proof of Rubin-T. Theorem

Let's suppose our "book-keeping" function told us to focus on $\nu(i) = k \leq i$. This means: focus on X_k and build $EW(P_i^k, \mathbb{Z}/p, \ell_k)$ above P_i^k.



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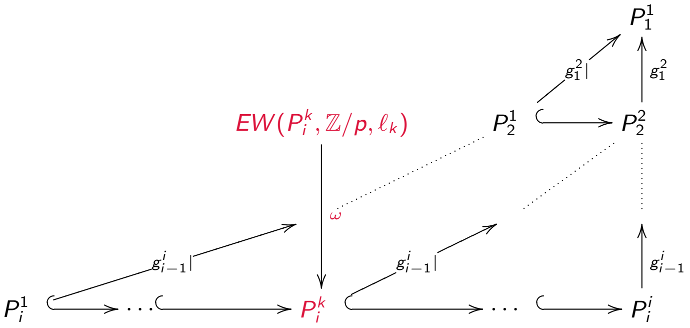
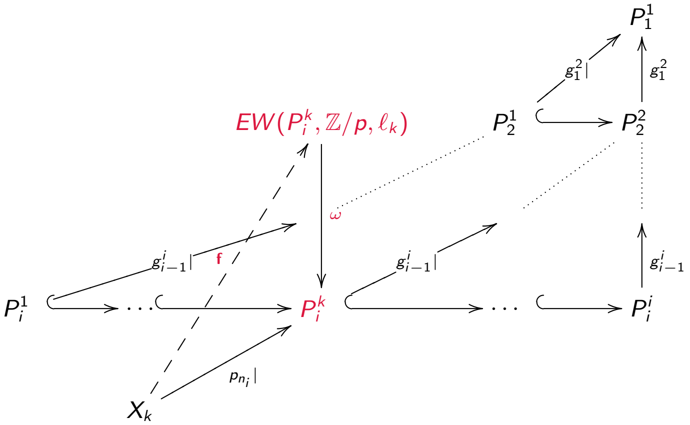


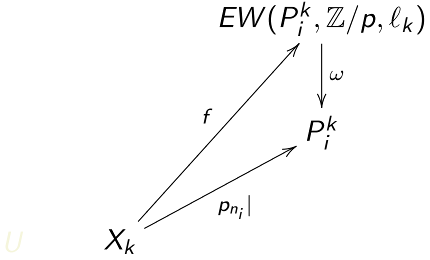
About the proof of Rubin-T. Theorem

Now there is an approximate lift $f : X_k \to EW(P_i^k, \mathbb{Z}/p, \ell_k)$ of $p_{n_i}| : X_k \to P_i^k$ (because $\dim_{\mathbb{Z}/p} X_k \leq \ell_k$).



About the proof of Rubin-T. Theorem

We can extend f over a nbhd U of X_k in Hilbert cube Q, then make this nbhd smaller so that on U maps p_{n_i} and $\omega \circ f$ are close.



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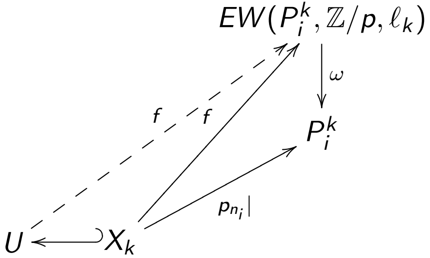

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Now you can pick n_{i+1}, as well as the polyhedra $P_{i+1}^{i+1} \supset P_{i+1}^{i} \supset \ldots \supset P_{i+1}^{k} \supset \ldots \supset P_{i+1}^{1}$ in $I^{n_{i+1}}$ so that they satisfy a number of technical properties, including $X_k \subset P_{i+1}^{k} \times Q_{n_{i+1}} \subset U$.

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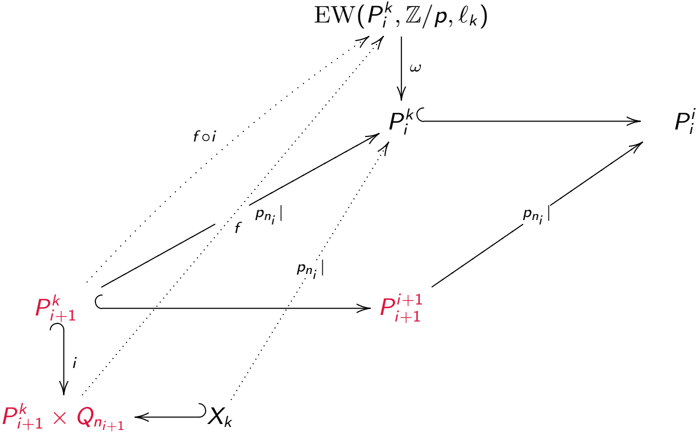

About the proof of Rubin-T. Theorem

Let $\widetilde{f}$ be a cellular approximation of $f \circ i$.

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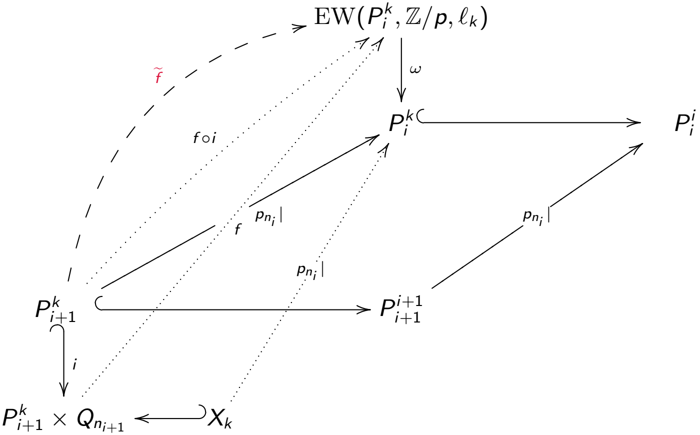
Let $\widetilde{f}$ be a cellular approximation of $f \circ i$. Because of our careful choices, we can extend $\omega\circ\widetilde{f}: P^k_{i+1} \to P^k_i$ to a map $\varphi: P^{i+1}_{i+1} \to P^i_i$, so that φ and $p_{n_i}|_{P^{i+1}_{i+1}}$ are very close. Finally, replace φ by its simplicial approximation $g^{i+1}_i: P^{i+1}_{i+1} \to P^i_i$.



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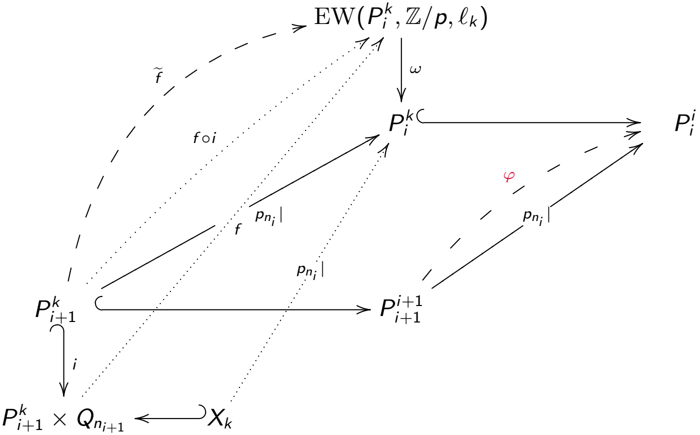

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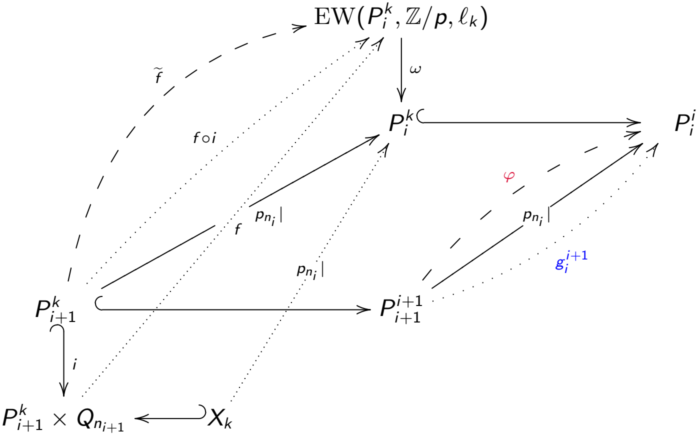
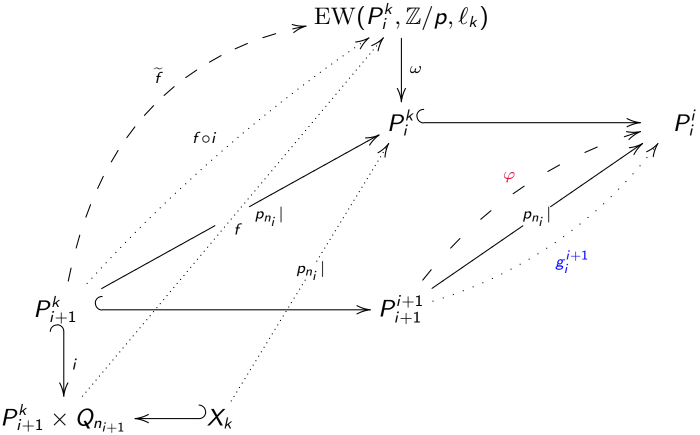


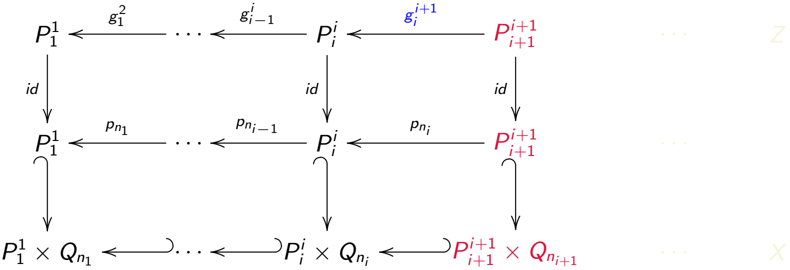
About the proof of Rubin-T. Theorem

Note that $g_i^{i+1}|_{P_{i+1}^k} : P_{i+1}^k \to P_i^k$ factors through $\mathrm{EW}(P_i^k, \mathbb{Z}/p, \ell_k)$ up to closeness/homotopy, and $g_i^{i+1} : P_{i+1}^{i+1} \to P_i^i$ is close to $p_{n_i}|_{P_{i+1}^{i+1}} : P_{i+1}^{i+1} \to P_i^i$.



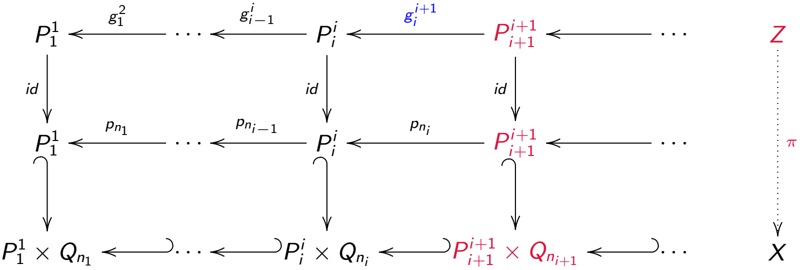
About the proof of Rubin-T. Theorem

This is how we get $P^{i+1}_{i+1} \subset I^{n_{i+1}}$ (together with $P^{i}_{i+1}, \ldots, P^{1}_{i+1}$), and the bonding map $g^{i+1}_{i} : P^{i+1}_{i+1} \to P^{i}_{i}$.

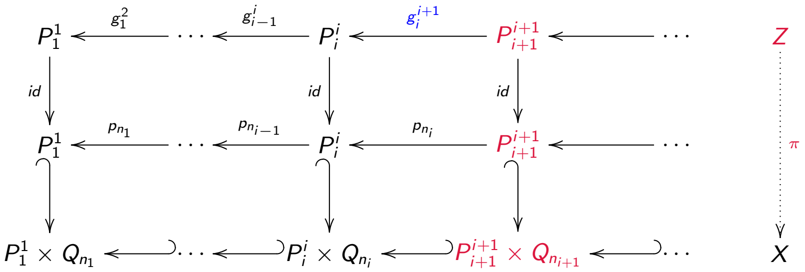


About the proof of Rubin-T. Theorem

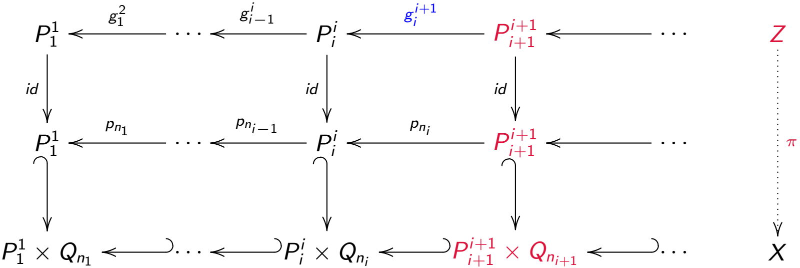
This is how we get $P_{i+1}^{i+1} \subset I^{n_{i+1}}$ (together with $P_{i+1}^{i}, \ldots, P_{i+1}^{1}$), and the bonding map $g_i^{i+1} : P_{i+1}^{i+1} \to P_i^i$.



We can define $\pi : Z \to X$ and π is continuous: from closeness of $g_i^{i+1} : P_{i+1}^{i+1} \to P_i^i$ and $p_{n_i} : P_{i+1}^{i+1} \to P_i^i$.

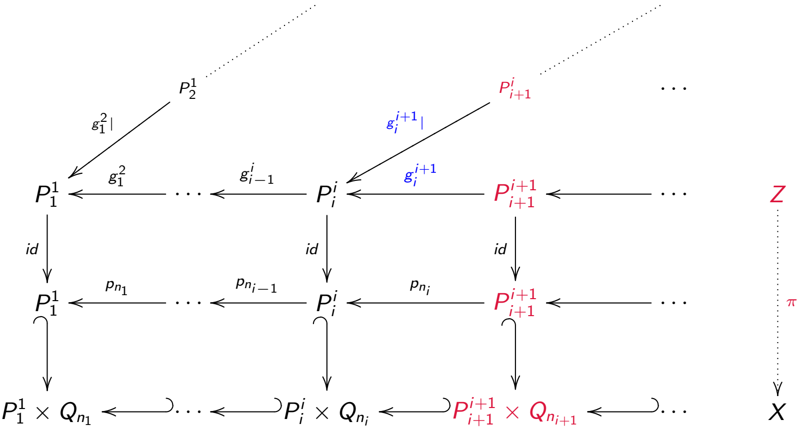


Cell-likeness of π: $\forall x \in X$, $\pi^{-1}(x)$ is the inverse limit of an inverse sequence $(P_{x,i}, g_i^{i+1}|)$ of contractible polyhedra.



About the proof of Rubin-T. Theorem

$\mathbb{Z}/p$-acyclicity of $\pi|_{Z_k} : Z_k \to X_k$: within each diagonal, infinitely many of $g_i^{i+1}|$ factor, up to homotopy, through an EW-complex.



THE END