

Trigonometric Functions

In this lesson, we will review the 6 trigonometric functions on a right triangle and the unit circle.

In Calculus, we measure angles in **radians** instead of **degrees**:

$$\pi \text{ radians} = 180^\circ$$

Example 1: Find the radian measure of 60° .

Solution:

$$60^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{3}$$

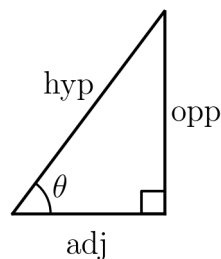
Example 2: How many degrees is $\frac{3\pi}{4}$ radians?

Solution:

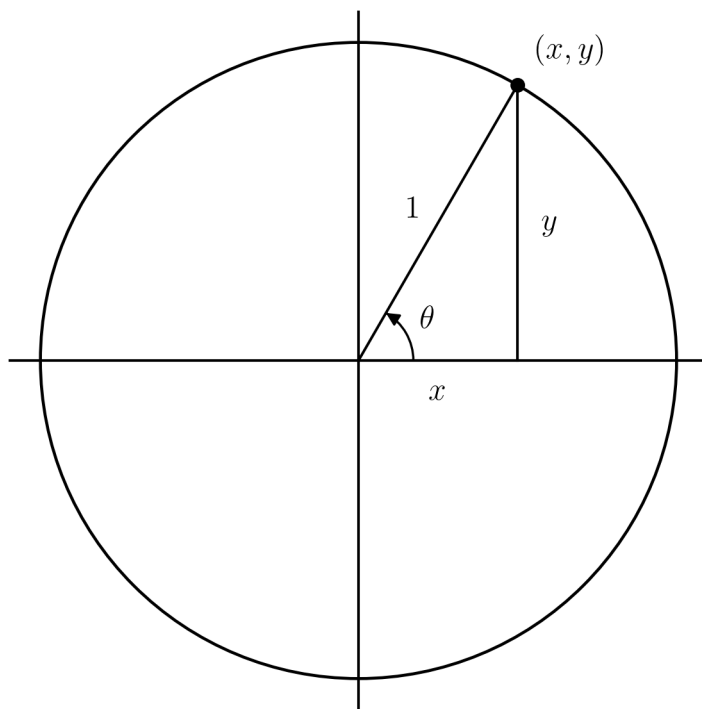
$$\frac{3\pi}{4} \cdot \frac{180^\circ}{\pi} = \frac{3}{4} \cdot 180^\circ = 135^\circ$$

Trigonometric Functions on a Right Triangle: For acute angles, we can define the trigonometric functions in terms of a right triangle.

$$\begin{array}{ll} \sin(\theta) = \frac{\text{opp}}{\text{hyp}} & \csc(\theta) = \frac{\text{hyp}}{\text{opp}} \\ \cos(\theta) = \frac{\text{adj}}{\text{hyp}} & \sec(\theta) = \frac{\text{hyp}}{\text{adj}} \\ \tan(\theta) = \frac{\text{opp}}{\text{adj}} & \cot(\theta) = \frac{\text{adj}}{\text{opp}} \end{array}$$



Trigonometric Functions on the Unit Circle: The unit circle has center $(0, 0)$ and radius 1. We measure angles from the positive x -axis. Positive is counterclockwise, and negative is clockwise.



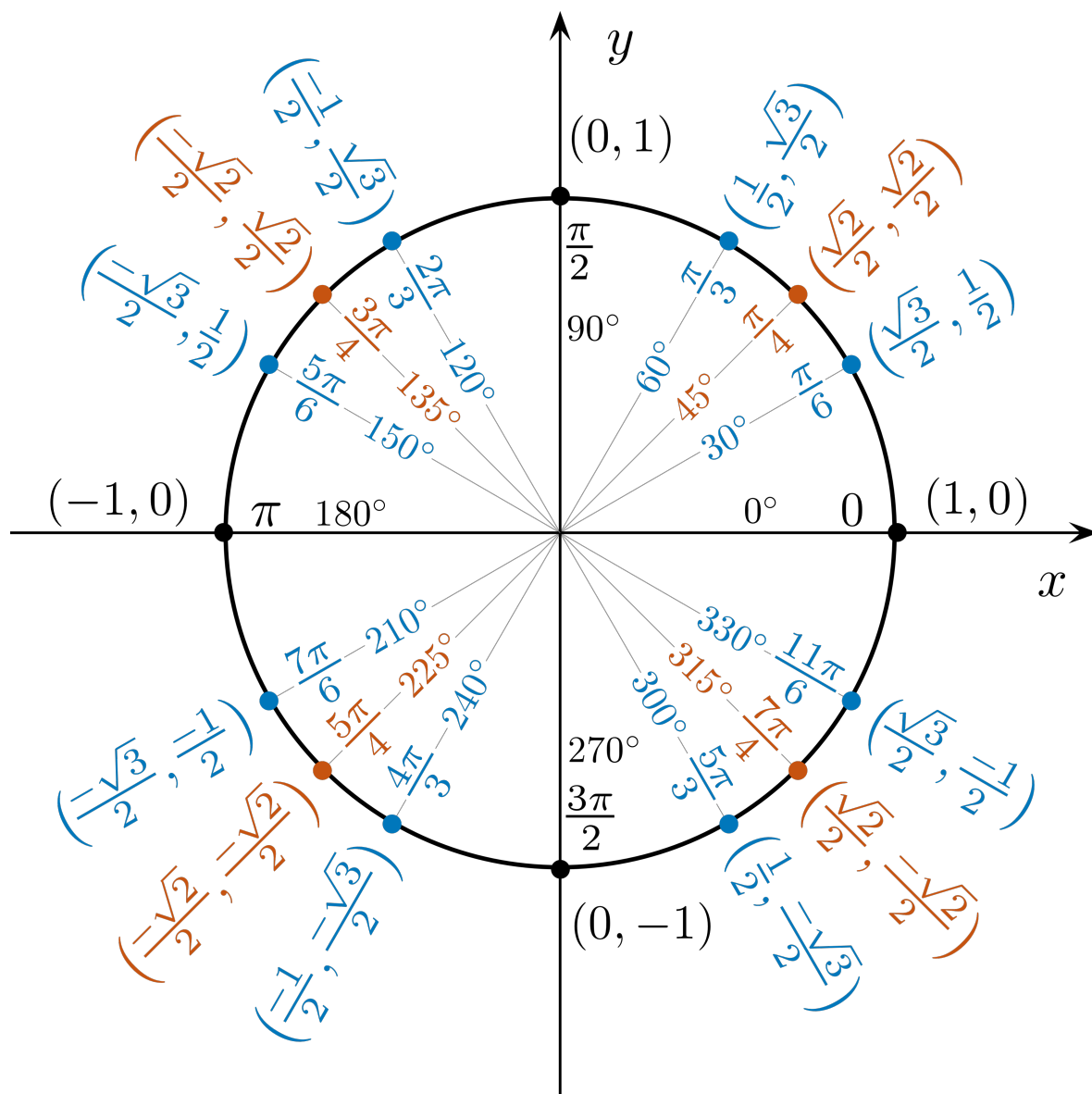
The trigonometric functions can be expressed in terms of the point (x, y) on the unit circle:

$$\sin \theta = y \qquad \csc \theta = \frac{1}{y}$$

$$\cos \theta = x \qquad \sec \theta = \frac{1}{x}$$

$$\tan \theta = \frac{y}{x} \qquad \cot \theta = \frac{x}{y}$$

Here is a unit circle with common angles labeled:



Example 3: Evaluate each trigonometric expression:

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\tan\left(\frac{5\pi}{4}\right) = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$$

$$\sec\left(\frac{\pi}{3}\right) = \frac{1}{\frac{1}{2}} = 2$$