Sigma Notation

In this lesson, we will explore sigma notation, and use summation formulas to evaluate sums.

When approximating the area under a curve with rectangles, we end up with long sums of terms. **Sigma notation** is a way to write sums more compactly. For example,

$$1 + 2 + 3 + 4 + 5 + 6 + 7 = \sum_{i=1}^{7} i$$

The symbol $\sum$ is the Greek letter sigma, which stands for sum. The index i begins at $i = 1$, increases by 1 each time, and ends at $i = 7$. The variable i is the index of summation, which is a placeholder and not part of the evaluated sum.

Example 1: Write $\displaystyle\sum_{k=1}^{5}(2k+1)$ in expanded form.

Example 2: Write the sum $0 + 1 + 4 + 9 + 16 + \ldots + 100$ in sigma notation.

Example 3: Write $\displaystyle\sum_{i=1}^{n}(3i-2)$ in expanded form.

We can factor out constants from a sum, or split a sum into two sums:

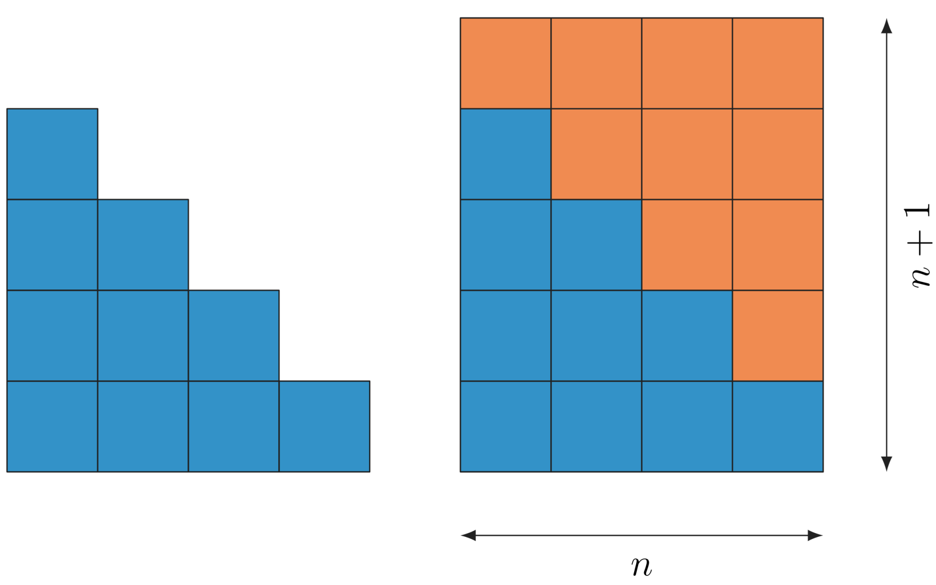
Theorem Properties of Summation: Let c be a constant and n a positive integer. Then:

$$\sum_{i=1}^{n} c\, a_i = c \sum_{i=1}^{n} a_i$$
$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$
$$\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$$

Summation formulas help us evaluate sums. For example, if we add the same constant c from $i = 1$ to n,

$$\sum_{i=1}^{n} c =$$

We can find $\sum_{i=1}^{n} i$ by drawing a staircase diagram:



The blue staircase has

Here are some common summation formulas:

Theorem **Summation Formulas:** Let n be a positive integer. Then:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Example 4: Evaluate $\displaystyle\sum_{i=1}^{n}(i^2 + 2i - 3)$.