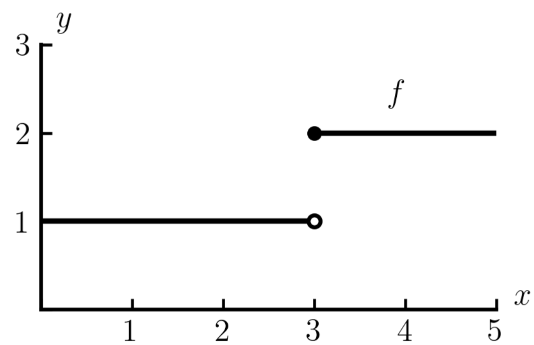
One-Sided Limits

In this lesson, we will explore one-sided limits, their relationship with two-sided limits, and practice finding one-sided limits from graphs.

Sometimes a limit does not exist, but we can still take limits from the left or right.



Limit from the left: $\lim_{x \to 3^-} f(x) = 1$

Limit from the right: $\lim_{x \to 3^+} f(x) = 2$

Two-sided limit: $\lim_{x \to 3} f(x)$ does not exist

Definition **Left-Sided Limit:** Let f be defined for all x near a with $x < a$. We say $\lim_{x \to a^-} f(x) = L$, if we can make $f(x)$ as close as we want to L by choosing x close enough to a with $x < a$.

Read Aloud As: "The limit of $f(x)$ as x approaches a from the left equals L."

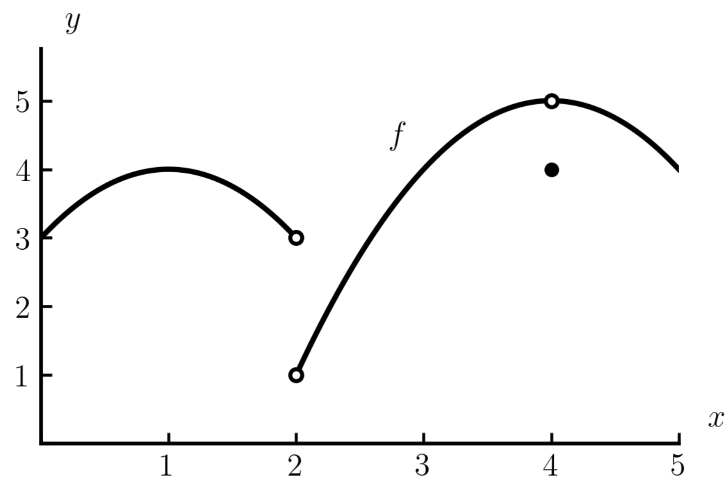
Definition **Right-Sided Limit:** Let f be defined for all x near a with $x > a$. We say $\lim_{x \to a^+} f(x) = L$, if we can make $f(x)$ as close as we want to L by choosing x close enough to a with $x > a$.

Read Aloud As: "The limit of $f(x)$ as x approaches a from the right equals L."

Theorem **One-Sided and Two-Sided Limits:** Let f be defined for all x near a, but possibly not at a. Then $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$.

Example 1: For the function f in the graph, find each value if they exist.

Solution:



$f(2)$ is undefined	$\lim_{x\to 2^-} f(x) = 3$	$\lim_{x\to 2^+} f(x) = 1$	$\lim_{x\to 2} f(x)$ DNE
$f(4) = 4$	$\lim_{x\to 4^-} f(x) = 5$	$\lim_{x\to 4^+} f(x) = 5$	$\lim_{x\to 4} f(x) = 5$