One-Dimensional Motion

We will use derivatives to find velocity and acceleration and solve problems involving one-dimensional motion.

We have seen how if s gives the position of an object at time t, then $v = s'$ gives the instantaneous velocity of the object at time t. Velocity can be positive or negative, depending on the direction the object is moving. The magnitude of velocity is called the speed, which is always positive. The instantaneous rate of change of velocity is called the acceleration.

Definition **Velocity, Speed, and Acceleration:** Suppose an object moves along a straight line with position function $s(t)$. The velocity, speed, and acceleration are given by:

Velocity: $v(t) = s'(t)$

Speed: $|v(t)|$

Acceleration: $a(t) = v'(t) = s''(t)$

Example 1: On Mars, a rock is dropped from a height of 100 feet. Its height in feet after t seconds is given by $s(t) = 100 - 6t^2$. Find the velocity, speed, and acceleration of the rock at time $t = 2$ seconds.

Solution: The velocity is $v(t) = s'(t) = \dfrac{d}{dt}(100 - 6t^2) = -12t$.

The acceleration is $a(t) = v'(t) = \dfrac{d}{dt}(-12t) = -12$.

When $t = 2$ seconds, the velocity is $v(2) = -12(2) = -24$ ft/s, the speed is $|v(2)| = |-24| = 24$ ft/s, and the acceleration is $a(2) = -12$ ft/s^2.

Example 2: An object moves horizontally along a straight line with position function $s(t) = t^3 - 6t^2 + 9t$. When is the object at rest? What is the acceleration at those times? When is the object moving right (positive direction) and left (negative direction)?

Solution: The velocity is $v(t) = s'(t) = \dfrac{d}{dt}(t^3 - 6t^2 + 9t) = 3t^2 - 12t + 9$.

The acceleration is $a(t) = v'(t) = \dfrac{d}{dt}(3t^2 - 12t + 9) = 6t - 12$.

The object is at rest when $v(t) = 0$. Solving the equation:

$$3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t-1)(t-3) = 0$$

we see the object is at rest at $t = 1$ and $t = 3$.

The acceleration at those times is $a(1) = 6(1) - 12 = -6$ ft/s^2 and $a(3) = 6(3) - 12 = 6$ ft/s^2.

To determine when the object is moving right and left, we need to find when $v(t) > 0$ and $v(t) < 0$. We can use a sign line to determine the sign of $v(t)$ in each interval.

The object is moving right when $t < 1$ and $t > 3$, and moving left when $1 < t < 3$.