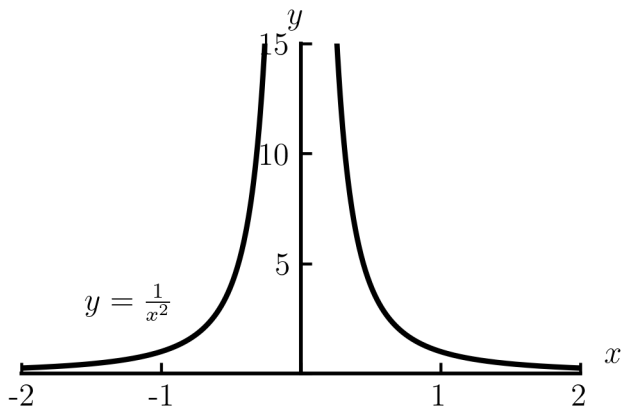
Infinite Limits

In this lesson, we will explore infinite limits and define vertical asymptotes in terms of limits.

What happens to $f(x) = \dfrac{1}{x^2}$ as x approaches 0 from either side?



x	$f(x)$
± 0.1	100
± 0.01	10,000
± 0.001	1,000,000

Definition **Infinite Limit:** Let f be defined for all x near a. We say $\lim\limits_{x \to a} f(x) = \infty$, if we can make $f(x)$ as large as we want by choosing x close enough to a. We say $\lim\limits_{x \to a} f(x) = -\infty$, if we can make $f(x)$ negative and as large in magnitude as we want by choosing x close enough to a.

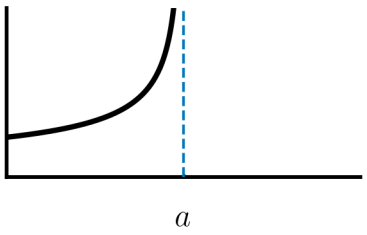
Read Aloud As: "The limit of $f(x)$ as x approaches a is infinity," or "The limit of $f(x)$ as x approaches a is negative infinity."

Careful: If $\lim\limits_{x \to a} f(x) = \infty$ or $\lim\limits_{x \to a} f(x) = -\infty$, the limit does not exist, since $f(x)$ does not approach a single value as x approaches a.

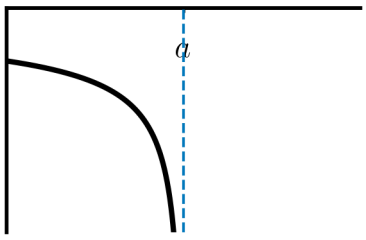
Definition **One-Sided Infinite Limit:** If we let f be defined for all x near a with $x < a$, we say $\lim_{x \to a^-} f(x) = \infty$, if we can make $f(x)$ as large as we like whenever x is sufficiently close to a with $x < a$.

Read Aloud As: "The limit of $f(x)$ as x approaches a from the left is infinity."

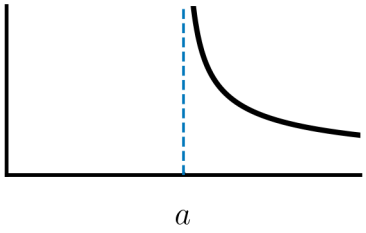
We can define $\lim_{x \to a^+} f(x) = \infty$, $\lim_{x \to a^-} f(x) = -\infty$, and $\lim_{x \to a^+} f(x) = -\infty$ similarly.



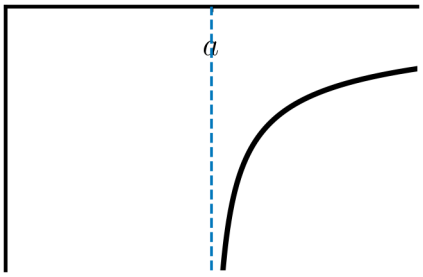
$\lim_{x \to a^-} f(x) = \infty$



$\lim_{x \to a^-} f(x) = -\infty$



$\lim_{x \to a^+} f(x) = \infty$



$\lim_{x \to a^+} f(x) = -\infty$

Definition Vertical Asymptote: The vertical line $x = a$ is called a vertical asymptote of f if $\lim_{x\to a} f(x) = \pm\infty$, $\lim_{x\to a^-} f(x) = \pm\infty$, or $\lim_{x\to a^+} f(x) = \pm\infty$.

Example 1: For $f(x) = \dfrac{-x}{(x-2)^2(x-4)}$, find the limits below.

Solution:

$$\lim_{x\to 2^-} f(x) = \infty, \qquad \lim_{x\to 2^+} f(x) = \infty, \qquad \lim_{x\to 2} f(x) = \infty.$$
$$\lim_{x\to 4^-} f(x) = \infty, \qquad \lim_{x\to 4^+} f(x) = -\infty, \qquad \lim_{x\to 4} f(x) \text{ DNE}$$