Differentials

In this lesson, we will introduce differentials, and use them to approximate changes in functions.

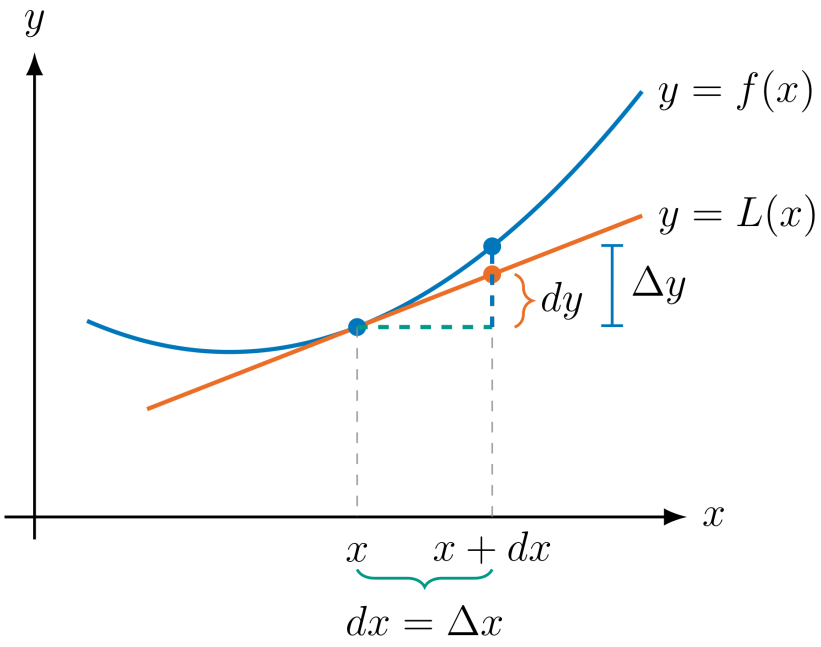
If $y = f(x)$ is differentiable at a, the linear approximation of f at a is

Let $\Delta x = x - a$ denote the change in x, and let $\Delta y = f(x) - f(a)$ denote the corresponding change in $y = f(x)$.

In the limit as $\Delta x \to 0$, this approximation gets better and better, and in the limit is exactly equal. This is called the differential form:

Now we will replace the fixed point a with the variable x. If f is differentiable at x, then

The quantities dx and dy are called **differentials**.



When x changes by $dx = \Delta x$,

- Actual change:
- Linear change:

Example 1: Use differentials to estimate how $f(x) = \dfrac{1}{x^2}$ changes as x increases from 2 to 2.1. Compare your estimate to the actual change.

Example 2: A sphere has radius 10 cm and is painted with a coat of paint 2 mm thick. Use differentials to estimate the volume of paint used.