Differentiability

We will show that if f is differentiable at a point, then f is continuous at that point. Then we will explore three types of points where functions are not differentiable.

Theorem Differentiability Implies Continuity: If f is differentiable at a, then f is continuous at a.

Proof: If $x \neq a$, then

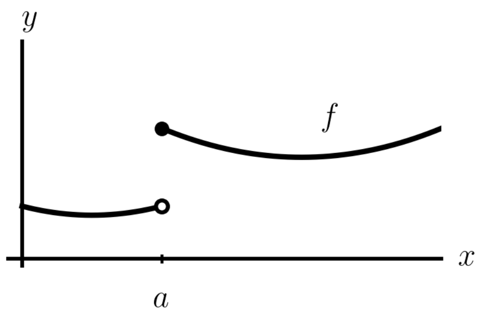
$$f(x) = \frac{f(x) - f(a)}{x - a} \cdot (x - a) + f(a).$$

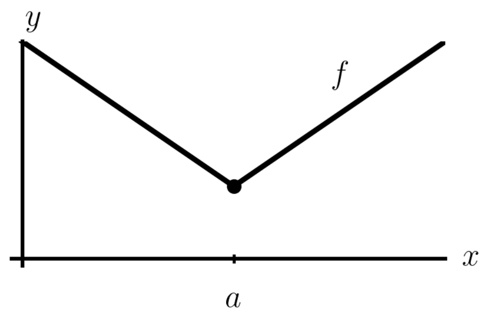
So:

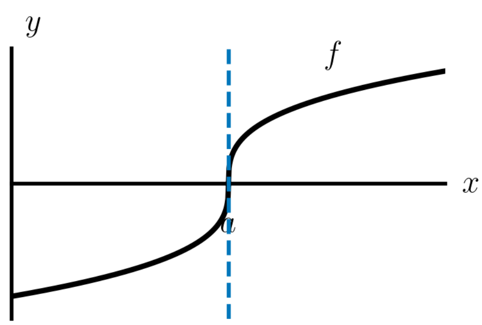
The contrapositive is also true: If f is not continuous at a, then f is not differentiable at a.

Careful: If f is continuous at a, then f may or may not be differentiable at a.

3 Types of Non-Differentiability: f is not differentiable at a if:







Example 1: For the function f in the graph, identify where f is not continuous and where f is not differentiable.