Complex Geometry

A Conference in Honor of
Domingo Toledo's 60th Birthday

University of Utah
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Abstracts

Daniel Allcock: A monstrous proposal

A series of coincidences suggests an appearance of the monster simple group in the deck group of a branched cover of a particular arithmetic quotient of complex hyperbolic 13-space, possibly with a moduli-space interpretation.

Jim Carlson: New Hodge theory for cubic threefolds

The moduli space of smooth cubic threefolds, like that of cubic surfaces, has a complex hyperbolic structure. That is, it can be realized as the quotient of the unit ball in a complex Euclidean space, minus a set of totally geodesic complex hyperplanes, modulo an arithmetic group. We discuss this construction and a natural partial compactification of it which is isomorphic to the ball modulo the group. Geometrically, the components added are (a) nodal cubics (b) the secant variety of a rational normal curve of degree four with a set of twelve points marked on it. The identification with a ball quotient involves two pieces of Hodge theory. First is the cyclic cover trick, which goes back to Picard and which was exploited by Deligne and Mostow in their study of moduli of points on the projective line and their relation to ball quotients. Second is a way of relating certain complex Hodge structures of weight four on a fourfold to other complex Hodge structures of weight one on an algebraic curve. *(Joint work with Domingo Toledo and Daniel Allcock).*

Bill Goldman: Toledo's invariant of surface group representations

(tentative abstract) First, a historical account of early work of the author and of Toledo's Math Scandinavica harmonic maps paper and how it led to the local rigidity of surface

groups in $U(1,1)$ in $U(n,1)$. This then led to Toledo's global rigidity on surface group representations in $U(n,1)$. I will also discuss connections with my work with Millson on higher dimensional local rigidity, which led into Corlette's thesis on global rigidity. I may also try to survey some of the work in complex hyperbolic geometry (Parker, Falbel, Gusevskii) on the Toledo invariant and (maybe if time allows) work of Bradlow/Garcia-Prada/Gothen/Mundet/Xia on Higgs bundles.

Luis Hernández: Almost-hermitian structures of minimal energy

Let (M, g) be a compact Riemannian manifold and J an orthogonal almost-complex structure on M. The energy of J is defined as

$$E(J) = \int_M \nabla\omega,$$

where ω is the Kähler form associated to J and g.

Reminiscent of what happens in Yang-Mills theory, we show this energy decomposes into pieces according to a certain U_n-representation (exactly two pieces when dim $M = 4$), and a certain linear combination of such pieces (the difference of the two, in dimension 4) turns out to be, not a topological invariant as in Yang-Mills theory, but a multiple of the total scalar curvature (and thus depending only on g) when the metric g happens to be conformally flat (of ASD in dimension 4).

We'll use this to give examples of minimal energy J , e.g. the almost-complex J given by Cayley multiplication on the round S^6, the usual hermitian structure on $S^3 \times S^1$, etc.

Misha Kapovich: Generalized triangle inequalities and their applications

Abstract. Everybody knows how to construct triangles in the Euclidean plane given their side-lengths which satisfy the familiar triangle inequalities. In this talk I will explain how to generalize this in the setting of nonpositively curved symmetric spaces and buildings, where the real-valued distance function is replaced by an appropriate vector-valued function. If the time permits I will explain the relation of this problem to the geometric invariant theory and theoretical computer science.

János Kollár: Holonomy groups of stable vector bundles

Abstract: We define the notion of holonomy group for a stable vector bundle F on a variety in terms of the Narasimhan–Seshadri unitary representation of its restriction to curves. Next we relate the holonomy group to the minimal structure group and to the decomposition of tensor powers of F. Finally we illustrate the principle that either the

holonomy is large or there is a clear geometric reason why it should be small. (Joint work with Balaji.)

Bruno Klingler: On the Andre-Oort conjecture

Abstract : The Andre-Oort conjecture describes the geometry of collection of special points on Shimura varieties : any irreducible component of the Zariski closure of a set of special points on a Shimura variety is conjectured to be a subvariety of Hodge type. I will explain a proof of this conjecture under the Generalized Riemann Hypothesis (joint work with Andrei Yafaev).

Yum-Tong Siu: Multiplier ideal sheaves and the finite generation of canonical rings

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Dennis Sullivan: Operations in the string spaces of a smooth manifold and compactified spaces of Riemann surfaces

There are canonical minimal energy area preserving flows on Riemann surfaces with input punctures where the fluid enters at given rates and output punctures where the flow exits at given rates.These flows are pictorially easy to analyze as the surface developes nodes. There are also versions on surfaces with boundary...

The combinatorics of the orbits touching rest points of the flows leads to two discussions.

The first is a cell decomposition of the open moduli space of Riemann surfaces (discussed first by Giddings and Wolpert in a physics oriented paper and independently by C.F. Bodigheimer in a more precise mathematical treatment). The open moduli space cell discussion extends naturally to the nodal compactification of moduli space using the above mentioned pictures.

The second discussion makes use of the combinatorics, in particular how the flow moves, splits and reconnects the orthogonal one manifolds to the flow, and regularized transversality in families of poly curves in a manifold to construct operations in the algebraic topology of the free loop space as well as in that of path spaces with boundary conditions. The regularization is a technical device which diffuses the objects acted upon using a measure on a finite dimensional space of diffeomorphisms constructed from a coordinate cover.

Anna Weinhard: Why you should maximize the Toledo invariant

The Toledo invariant associates a real number to every homomorphism of the fundamental group of a (closed) Riemann surface into a semisimple Lie group of Hermitian type. The Toledo invariant is locally constant and bounded. Homomorphisms which realize the maximal possible value of the Toledo invariant were studied around 15 years ago by Goldman, Toledo and Hernandez. They showed that in many cases homomorphisms with maximal Toledo invariant have nice geometric properties. I will present recent work with Burger and Iozzi on homomorphisms of fundamental groups of (not necessarily closed) Riemann surfaces with maximal Toledo invariants extending their results. Our approach owes a lot to Toledo's treatment of the problem when the Lie group of Hermitian type is the isometry group of a complex hyperbolic space.