Title. Density functions for families of graded ideals.

Abstract. In the first part of the talk we discuss Hilbert-Kunz density function and its applications for positive characteristic invariants, namely Hilbert-Kunz multiplicities and F-thresholds.

Next, based on a joint work with Suprajo Das and Sudeshna Roy, we introduce the *adic* and *saturated* density functions. These were introduced to give numerical characterizations for integral dependence of ideals in a graded set up, where we recall that two ideals $I \subset J$ in a commutative Noetherian ring are *integrally dependent* if they have the same integral closure.

Attempts to give a numerical characterization for ideals which might not necessarily be of finite colength led to numerical invariants like j-multiplicity, ε-multiplicity which require (even in graded set up) computing the invariant at several localizations, hence not readily amenable to computations. Also there exists a notion of multiplicity sequence which gives a numerical characterization of integral dependence.

As an application of these density functions we show that any of the multiplicities, namely, the polar multiplicities, the ε-multiplicities or the j-multiplicities of the truncated ideals $I[Y]_{\geq c}$ and $J[Y]_{\geq c}$ in $R[Y]$ characterizes the integral dependence of I and J. A novelty of this approach is that it does not involve localization and only requires checking computable and well-studied invariants like Hilbert-Samuel multiplicities.