The following three formulas may be helpful in solving problems from question 1.

- Sum of the first \( n \) terms of an arithmetic series with first term \( a_1 \) and common difference \( d \):
  \[
  \frac{n}{2} (2a_1 + (n - 1)d)
  \]

- Sum of the first \( n \) terms of a geometric series with first term \( a_1 \) and common ratio \( r \):
  \[
  \frac{a_1(1 - r^n)}{1 - r}
  \]

- Sum of an infinite geometric series:
  \[
  \sum_{n=1}^{\infty} a r^{n-1} = \frac{a}{1 - r}
  \]

1. Evaluate each of the following, if possible. If the series diverges, explain why.

\[
\sum_{k=1}^{50} (2k - 1)
\]

\[
\sum_{j=1}^{\infty} 1.5(2)^{j-1}
\]

\[
\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^{k-1}
\]

2. Given that \( x=2 \) is a zero of the polynomial function \( f(x) = x^3 - 2x^2 + 4x - 8 \), find all the other real and complex roots of \( f(x) \).

3. Given \( f(x) = x^3 \), complete the following tasks:
   a.) Graph \( f(x) \) and label three points.
   b.) List the transformations (in order) for the graph of \( f(x) \) to obtain the graph of \( g(x) = -(x-1)^3 - 2 \).
   c.) Graph \( g(x) \) and label three points.
4. The graphs of functions \( f(x), g(x) \) are below. Use them to compute the indicated values.

![Graphs of f(x) and g(x)]

a.) \((f \circ g)(1)\)

b.) \((g \circ f)(2)\)

c.) \((g \circ g)(0)\)

d.) \((f \circ f)(-1)\)

5. Solve the following logarithmic equations. Eliminate any extraneous solutions.

a.) \(\log_4 x + \log_4(x - 6) = 2\)

b.) \(\log_3 4x - \log_3(x + 2) = 1\)

6. Evaluate:  
a.) \(\ln e^4\)  
b.) \(\log_{10}\left(\frac{1}{100}\right)\)

7. Write as a single logarithm: \(2 \log_4 x + 5 \log_4 y - \frac{1}{2} \log_4 z\).

8. Solve the inequality \(\frac{4}{x+1} \geq 1\) and give your answer in interval notation.

9. For the function \(f(x) = x^2 - 2x\),

a.) Find the x-intercepts, if any exist.

b.) Find the y-intercept, if it exists.

c.) Find the vertex.

d.) Sketch a graph of \(y = f(x)\).

10. Find the inverse of the function \(f(x) = (x - 1)^3 - 2\).

11. Classify each of the following sequences as arithmetic, geometric, or neither.

a.) \(17, 9, 1, -7, -15, ...\)

b.) \(2, -4, 8, -16, 32, ...\)

c.) \(\frac{8}{3}, 4, 6, 9, \frac{27}{2}, ...\)

d.) \(\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, ...\)

e.) \(-2, 6, -10, 14, -18, ...\)
12. Find a formula for the 90th term of the sequence $-7, -4, -1, 2, 5, ...$

13. Is the function shown in the graph invertible? Why or why not?
Solutions to MATH 1050 Sample Final Exam

1.) Using the first formula from the exam,
\[ \sum_{k=1}^{50} (2k-1) = \frac{50}{2} (2(1)+49(2)) \]
\[ = \frac{50}{2} (100) \]
\[ = 50(50) \]
\[ = 2,500 \]
\[ \sum_{j=1}^{8} 1.5(2)^{j-1} \] diverges since \( 2 > 1 \).

Using the third formula from the exam,
\[ \sum_{k=1}^{80} \left(\frac{1}{3}\right)^{k-1} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2} \]

2.) Since 2 is a zero of \( x^3-2x^2+4x-8 \), \( x-2 \) is a factor of \( x^3-2x^2+4x-8 \). Using long division:

\[
\begin{array}{c|cccc}
\times-2 & x^2 & +4 \\
\hline
x-2 & x^3 & -2x^2 & +4x-8 \\
\hline
\multicolumn{4}{r}{x^3-2x^2} \\
\hline
\multicolumn{4}{r}{4x-8} \\
\hline
\multicolumn{4}{r}{4x-8} \\
\hline
\multicolumn{4}{r}{0} \\
\end{array}
\]

so \( x^3-2x^2+4x-8 \) equals \( (x-2)(x^2+4) \).
3. a.)

b.) \( x^3 \rightarrow -(x-1)^3 - 2 \)

\[ \text{flip over } x\text{-axis} \quad \text{right 1} \quad \text{down 2} \]

To graph \( g(x) = -(x-1)^3 - 2 \), start with the graph of \( f(x) = x^3 \), shift it right 1, flip over \( x\)-axis, and shift down 2.

c.)

4. a.) \( f(g(1)) = f(3) = 1 \)

b.) \( g(f(2)) = g(0) = -3 \)

c.) \( g(g(0)) = g(-3) = 3 \)

d.) \( f(f(-1)) = f(-3) = -5 \)
5.) a) \[ 2 = \log_4(x) + \log_4(x-6) \]
    \[ = \log_4(x(x-6)) \]
    \[ = \log_4(x^2 - 6x) \]
so \[ 4^2 = x^2 - 6x. \] Thus, \[ x^2 - 6x - 16 = 0. \]
The roots of \[ x^2 - 6x - 16 \] are 8 and -2, however, -2 is an extraneous solution since if \[ x = -2, \] then \[ \log_4(x) = \log_4(-2) \]
and we can't take a logarithm of a negative number.

b) \[ 1 = \log_3(4x) - \log_3(x+2) = \log_3\left(\frac{4x}{x+2}\right) \]
so \[ 3^1 = \frac{4x}{x+2}. \] Thus, \[ 3(x+2) = 4x, \]
so \[ 6 = x. \]

6.) \[ \ln(e^4) = 4 \quad \text{and} \quad \log_{10}\left(\frac{1}{100}\right) = \log(10^{-2}) = -2. \]
7.) \[ 2 \log_4 x + 5 \log_4 y - \frac{1}{2} \log_4 z \]
\[ = \log_4 x^2 + \log_4 y^5 - \log_4 \sqrt{z} \]
\[ = \log_4 \left( \frac{x^2 y^5}{\sqrt{z}} \right) \]

8.) \( \frac{4}{x+1} \) is \( \frac{1}{2} \) shifted left by 1, and scaled vertically by 4:

\[ \frac{4}{x+1} \geq 1 \text{ where the graph of } \frac{4}{x+1} \text{ is at or above } y=1. \]
\( \frac{4}{x+1} \text{ is at } y=1 \text{ where } \frac{4}{x+1} = 1, \)
so \( x+1 = 4 \text{ and } x=3. \)

From the graph, we see the answer is \((-1, 3]\).
9.) a.) $x^2-2x = x(x-2)$ has roots 0 and 2. These are the x-intercepts.

b.) The y-intercept is $(0)^2-2(0)=0$.

c.) The vertex is at $x=1$, the midpoint of the roots. Thus, $y=(1)^2-2(1)=-1$ so the vertex is at $(x,y)=(1,-1)$.

d.)

10.) Replace $f(x) = (x-1)^3 - 2$ with $x=(f^{-1}(x) - 1)^3 - 2$, and solve for $f^{-1}(x)$:

$x+2=(f^{-1}(x)-1)^3$, so $\sqrt[3]{x+2} = f^{-1}(x)-1$, and $f^{-1}(x) = \sqrt[3]{x+2} + 1$. 
11.) a.) Arithmetic, subtracting 8.
    b.) Geometric, multiplying $-2$.
    c.) Geometric, multiplying $\frac{3}{2}$.
    d.) Arithmetic, subtracting $\frac{2}{3}$.
    e.) Neither
    f.) Geometric, multiplying $3^2$.

12.) $-7 + 90(3) = 263$

13.) It's not invertible. It doesn't pass the horizontal line test.