Exact Asymptotic Speed of a Traveling Wave with Large Noise

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Traveling waves are a widespread phenomenon in physical systems. The Fisher-Kolmogorov-Petrovskii-Piscuinov (FKPP) equation is one of the simplest equations exhibiting traveling waves. Since physical systems often experience random disturbances, there is interest in the study of traveling waves in the presence of noise. We study solutions $u_t(x)$ to the following stochastic FKPP equation:

$$\partial_t u = \partial_x^2 u + u(1-u) + \sigma\sqrt{u(1-u)}\dot{W}(t,x)$$

where $\dot{W}(t,x)$ is two-parameter white noise. Here $x \in \mathbf{R}$, and we assume that $u_0(x) = \mathbf{1}(x \le 0)$.

One can define the right hand edge $R(u_t)$ of the solution, and we consider the speed

$$V(\sigma) = \lim_{t\to\infty} \frac{R(u_t)}{t}$$

There has already been work on the asymptotics of $V(\sigma)$ for σ near 0. For large σ, Conlon and Doering gave the lower bound

$$\liminf_{\sigma\to\infty} \sigma^2 V(\sigma) \ge 2.$$

We give a corresponding upper bound,

$$\limsup_{\sigma\to\infty} \sigma^2 V(\sigma) \le 2$$

which shows that $\lim_{\sigma\to\infty} \sigma^2 V(\sigma) = 2$.