1. (15 points) Vector Basics

For the vectors

\[
\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}
\]

answer the following, or explain why the question does not make sense:

(a) (3 points) \(2\mathbf{a} + 3\mathbf{c} = \)
\[
a = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad c = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}
\]

(b) (3 points) \(|a| =

(c) (2 points) What are the components of a unit vector in the same direction as \(a\)?
\[ a = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad c = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \]

(d) (4 points) \( b \cdot c = \)

(e) (3 points) \( a \cdot b \cdot c = \)
2. (10 points) Matrix Basics

For the matrices

\[ A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 1 & 5 \\ 4 & 4 & 2 \\ 1 & 0 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \]

answer the following, or explain why the question does not make sense:

(a) (3 points) \( A + C = \)
\[ A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 1 & 5 \\ 4 & 4 & 2 \\ 1 & 0 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \]

(b) (4 points) \(CB = \)

(c) (3 points) \(BC = \)
3. (15 points) Elimination Issues

(a) (5 points) For what value of $a$ in the system of equations below does elimination fail to produce a unique solution?

\[
\begin{align*}
3x + 2y &= 10 \\
6x + ay &= b
\end{align*}
\]

(b) (5 points) Given the determined value of $a$, for what value of $b$ are there an infinite number of solutions?

(c) (5 points) For the determined values of $a$ and $b$ what are two distinct solutions?
4. (20 points) Systems of Equations

Use elementary row operations to convert the system of equations

\[
\begin{align*}
2x & + 3y & + 3z & = 3 \\
6x & + 6y & + 12z & = 13 \\
12x & + 9y & - z & = 2
\end{align*}
\]

into upper-triangular form, and then use back-substitution to solve for the variables \(x, y, z\). Be sure to show all your work.
5. (15 points) *Inverting a Matrix*

Find the inverse of the matrix

\[
A = \begin{pmatrix}
1 & 1 & 1 \\
3 & 5 & 4 \\
3 & 6 & 5
\end{pmatrix}
\]
6. (15 points) *LDU Decomposition*

Find the *LDU* decomposition of the matrix

\[ A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{pmatrix} \]
7. (10 points) **Symmetric Products**
For the matrix

\[
R = \begin{pmatrix}
1 & 2 & 3 \\
2 & 4 & 0
\end{pmatrix}
\]

(a) (4 points) What is the transpose \( R^T \)?

(b) (4 points) What is the symmetric product \( R^T R \)?

(c) (2 points) Does \( R^T R = RR^T \)?