1 Section 2.3 - Elimination Using Matrices

2.3.1 Write down the 3 by 3 matrices that produce these elimination steps:

(a) $E_{21}$ subtracts 5 times row 1 from row 2.
(b) $E_{32}$ subtracts $-7$ times row 2 from row 3.
(c) $P$ exchanges rows 1 and 2, then rows 2 and 3.
2.3.2 In Problem 1, applying $E_{21}$ and then $E_{32}$ to $b = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ gives

$$E_{32}E_{21}b = \text{______________}.$$ 

Applying $E_{32}$ before $E_{21}$ gives

$$E_{21}E_{32}b = \text{______________}.$$ 

When $E_{32}$ comes first, row ____________ feels no effect from row ____________.
2.3.3 Which three matrices $E_{21}, E_{31}, E_{32}$ put $A$ into triangular form $U$?

\[
A = \begin{pmatrix}
1 & 1 & 0 \\
4 & 6 & 1 \\
-2 & 2 & 0
\end{pmatrix}
\quad \text{and} \quad E_{32}E_{31}E_{21}A = U.
\]
2.3.7 Suppose $E$ subtracts 7 times row 1 from row 3.

(a) To invert that step you should ____________ 7 times row ____________ to row ____________.

(b) What “inverse matrix” $E^{-1}$ takes the reverse step (so $E^{-1}E = I$)?

(c) If the reverse step is applied first (and then $E$) show that $EE^{-1} = I$. 

2.3.17 The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4)$ and $(2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns $(a, b, c)$. 
2 Section 2.4 - Rules for Matrix Operations

2.4.1 \( A \) is a 3 by 5, \( B \) is a 5 by 3, \( C \) is a 5 by 1, and \( D \) is 3 by 1. All entries are 1. Which of these matrix operations are allowed, and what are the results

\[
\begin{align*}
BA & \quad AB & \quad ABD & \quad DBA & \quad A(B + C).
\end{align*}
\]
2.4.2 What rows or columns or matrices do you multiply to find

(a) the third column of $AB$?
(b) the first row of $AB$?
(c) the entry in row 3, column 4 of $AB$?
(d) the entry in row 1, column 1 of $CDE$?
2.4.13 Which of the following matrices are guaranteed to equal \((A - B)^2\):

\[
\begin{aligned}
& A^2 - B^2, \\
& (B - A)^2, \\
& A^2 - 2AB + B^2, \\
& A(A - B) - B(A - B), \\
& A^2 - AB - BA + B^2?
\end{aligned}
\]
2.4.14 True or false:

(a) If $A^2$ is defined then $A$ is necessarily square.
(b) If $AB$ and $BA$ are defined then $A$ and $B$ are square.
(c) If $AB$ and $BA$ are defined then $AB$ and $BA$ are square.
(d) If $AB = B$ then $A = I$. 
2.4.32 (*Very important*) Suppose you solve $Ax = b$ for three special right sides $b$:

$Ax_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Ax_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $Ax_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

If the three solutions $x_1, x_2, x_3$ are the columns of a matrix $X$, what is $A$ times $X$?