Math 1010 - Exam 1

University of Utah

Fall 2009

Name: Solutions
1. Draw a real number line below, and plot and label the solutions to the following arithmetic problems. (10 points total)

\[ -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

(a) \( 3 + 2 = ? \) \hspace{2cm} (2 points)
\[ 3 + 2 = 5 \]

(b) \( -1 - (-3) = ? \) \hspace{2cm} (3 points)
\[ -1 - (-3) = 2 \]

(c) \( (-2) \times 3 = ? \) \hspace{2cm} (2 points)
\[ (-2) \times 3 = -6 \]

(d) \( \frac{2}{5} + \frac{3}{2} = ? \) \hspace{2cm} (3 points)
\[ \frac{2}{5} + \frac{3}{2} = \frac{4}{10} + \frac{15}{10} = \frac{19}{10} \]
2. Evaluate the following expressions. (15 points total)

(a) \( \frac{2}{3} + \frac{1}{5} \) (3 points)
\[
\frac{2}{3} + \frac{1}{5} = \frac{17}{15} + \frac{16}{15} = \frac{33}{15} = \frac{11}{5}.
\]

(b) \((-3)^3\) (3 points)
\[
(-3)^3 = -3 \times -3 \times -3 = 9 \times -3 = -27.
\]

(c) \(|-2| + (-|-4|)\) (3 points)
\[
|-2| + (-|-4|) = 2 + (-4) = -2.
\]

(d) \(\frac{3}{5} \div \frac{4}{3}\) (3 points)
\[
\frac{3}{5} \div \frac{4}{3} = \frac{3}{5} \times \frac{3}{4} = \frac{9}{20}.
\]

(e) \(21 - 5(7 - 5)\) (3 points)
\[
21 - 5(7 - 5) = 21 - 5(2) = 21 - 10 = 11.
\]
3. What property of real numbers is exemplified in the following expression: (5 points)

\[ a(b + c) = ab + ac \]

The distributive law of real numbers.

4. Simplify the following algebraic expressions. (15 points total)

(a) \( 8x - 5x + 7x \) (3 points)

\[ 8x - 5x + 7x = 10x. \]

(b) \( 3x^2 - 7 + 2x + 5x^2 + 11x - 3 \) (4 points)

\[ 3x^2 - 7 + 2x + 5x^2 + 11x - 3 = (3x^2 + 5x^2) + (11x + 2x) + (-7 + -3) \]
\[ = 8x^2 + 13x - 10. \]

(c) \( 8(z^3 - 4z^2 + 2) \) (3 points)

\[ 8(z^3 - 4z^2 + 2) = 8z^3 - 32z^2 + 16. \]

(d) \( x(x^2 + 3) - 3(x + 4) \) (5 points)

\[ x(x^2 + 3) - 3(x + 4) = x^3 + 3x - 3x - 12 = x^3 - 12. \]
5. Evaluate the following expressions for the specified values of the variable(s). If not possible, state the reason. (10 points total)

(a) $3y^2 + 10$
   i. $y = -2$  
   
   \[ 3(-2)^2 + 10 = 22 \]  
   
   (2 points)
   ii. $y = \frac{1}{2}$
   
   \[ 3\left(\frac{1}{2}\right)^2 + 10 = \frac{3}{4} + 10 = 10\frac{3}{4} \text{ or } 43 \frac{3}{4} \]  
   
   (3 points)

(b) $\frac{x}{x - y}$
   i. $x = 0, y = 10$
   
   \[ \frac{0}{0 - 10} = \frac{0}{-10} = 0. \]  
   
   (2 points)
   ii. $x = 3, y = 3$
   
   \[ \frac{3}{3 - 3} = \frac{3}{0} \]  
   
   which is undefined, as we cannot divide by zero. Therefore, evaluating the expression at $x = 3$ and $y = 3$ is not possible.


6. Find the value of $x$ that satisfies the given linear equation. (10 points total)

(a) $8x - 10 = 0$  

$8x - 10 = 0$

$\rightarrow 8x = 10$

$\rightarrow x = \frac{10}{8} = \frac{5}{4}$.

(b) $6(x + 2) = 30$

$6(x + 2) = 30$

$\rightarrow 6x + 12 = 30$

$\rightarrow 6x = 18$

$\rightarrow x = 3$

7. Solve the following percentage problems. (10 points total)

(a) What is 250% of 32?  

$2.5 \times 32 = 80$.

(b) What is 4% of 500?  

$.04 \times 500 = 20$. 
8. A restaurant sells a bottle of wine for $25 and paid $15 for the bottle.

(a) What is the markup? (3 points)

\[ 25 - 15 = 10. \]

(b) What is the markup rate? (5 points)

\[ \frac{10}{15} = \frac{2}{3} \text{ or } .66\overline{6} \text{ or } 66.\overline{6}\% . \]

9. Using the formula:

\[ F = \frac{9}{5}C + 32 \]

and given the fact that water freezes when \( C = 0^\circ \) and boils when \( C = 100^\circ \) calculate: (7 points total)

(a) The temperature in Fahrenheit at which water freezes.

(2 points)

\[ F = \frac{9}{5}(0) + 32 = 32^\circ F \]

(b) The temperature in Fahrenheit at which water boils.

(5 points)

\[ F = \frac{9}{5}(100) + 32 = 180 + 32 = 212^\circ F . \]
10. Solve the following inequality and sketch the solution on the real number line. (10 points)

\[ 3x - 11 > -x + 7. \]

\[
\begin{align*}
3x - 11 &> -x + 7 \\
\rightarrow 3x &> -x + 18 \\
\rightarrow 4x &> 18 \\
\rightarrow x &> \frac{9}{2}.
\end{align*}
\]