5.4 Factoring by Grouping and Special Forms

Write the number as a product of prime factors.

5.4.1: 6
\[ 2 \cdot 3 \]

5.4.4: 12
\[ 2 \cdot 2 \cdot 3 \]

5.4.5: 30
\[ 2 \cdot 3 \cdot 5 \]

5.4.8: 54
\[ 2 \cdot 3 \cdot 3 \cdot 3 \]

Find the greatest common factor of the expressions.

5.4.9: 16, 24
\[ 8 \]

5.4.15: 3x^2, 12x
\[ 3x \]

5.4.13: x^3, x^4
\[ x^3 \]

5.4.18: 9x^3y, 24xy^2
\[ 3xy \]

Factor out the greatest common monomial factor. (some of the polynomials have no common monomial factors.)

5.4.21: 4x + 4
\[ 4(x+1) \]

5.4.25: 24t^2 - 36
\[ 12(2t^2 - 3) \]

5.4.28: \( y^2 - 5y \)
\[ y(y - 5) \]

5.4.31: 11u^2 + 9
\[ \text{No common factor other than 1} \]

Factor a negative real number out of the polynomial and then write the polynomial factor in standard form.

5.4.41: 7 - 14x
\[ -7(2x - 1) \]

Factor the expression by factoring out the common binomial factor.
5.4.55: \(2y(y - 4) + 5(y - 4)\)

\[
(y - 4)(2y + 5)
\]

5.4.59: \(2(7a + 6) - 3a^2(7a + 6)\)

\[
(7a + 6)(2 - 3a^2)
\]

5.4.64: \((3x + 7)(2x - 1) + (x - 6)(2x - 1)\)

\[
(2x - 1)(4x + 1)
\]

Factor the polynomial by grouping.

5.4.65: \(x^2 + 25x + x + 25\)

\[
(x + 25)(x + 1)
\]

5.4.67: \(y^2 - 6y + 2y - 12\)

\[
(y - 6)(y + 2)
\]

5.4.69: \(x^3 + 2x^2 + x + 2\)

\[
(x + 2)(x^2 + 1)
\]

5.4.71: \(3a^3 - 12a^2 - 2a + 8\)

\[
(a-4)(3a^2 - 2)
\]

5.4.75: \(5x^3 - 10x^2y + 7xy^2 - 14y^3\)

\[
(x-2y)(5x^2 + 7y^2)
\]

Factor the difference of two squares.

5.4.77: \(x^2 - 9\)

\[
(x+3)(x-3)
\]

5.4.79: \(1 - a^2\)

\[
(1+a)(1-a)
\]
5.4.80: $16 - b^2$

$$\begin{align*}
(4 + b)(4 - b)
\end{align*}$$

5.4.93: $(x - 1)^2 - 16$

$$\begin{align*}
(x + 3)(x - 5)
\end{align*}$$

5.4.82: $9z^2 - 36$

$$\begin{align*}
(3z + 6)(3z - 6)
\end{align*}$$

5.4.95: $81 - (z + 5)^2$

$$\begin{align*}
(14 + z)(4 - z)
\end{align*}$$

5.4.85: $4z^2 - y^2$

$$\begin{align*}
(2z + y)(2z - y)
\end{align*}$$

Factor the sum or difference of cubes.

5.4.99: $x^3 - 8$

$$\begin{align*}
(x - 2)(x^2 + 2x + 4)
\end{align*}$$

5.4.103: $8t^3 - 27$

$$\begin{align*}
(2t - 3)(4t^2 + 6t + 9)
\end{align*}$$

Factor the polynomial completely.

5.4.111: $8 - 50x^2$

$$\begin{align*}
2(2 + 5x)(2 - 5x)
\end{align*}$$

5.4.115: $y^4 - 81$

$$\begin{align*}
(y - 3)(y + 3)(y^2 + 9)
\end{align*}$$

5.4.136: Chemical Reaction The rate of change of a chemical reaction is given by $kQx - kx^2$, where $Q$ is the amount of the original substance, $x$ is the amount of substance formed, and $k$ is a constant of the proportionality. Factor this expression.

$$\begin{align*}
kx(Q - x)
\end{align*}$$

5.4.138: Farming A farmer has enough fencing to construct a rectangular pig pen that encloses an area given by $32w - w^2$, where $w$ is the width (in feet) of the pen. Use factoring to find the length of the pen in terms of $w$.

$$\begin{align*}
32w - w^2 = (32 - w)w
\end{align*}$$

The length of the pen is $32 - w$.
Factor the perfect square trinomial.

5.5.1: \( x^2 + 4x + 4 \)

\[
(x+2)^2
\]

5.5.10: \( x^2 - 14xy + 49y^2 \)

\[
(x-7y)^2
\]

5.5.5: \( 25y^2 - 10y + 1 \)

\[
(5y-1)^2
\]

5.5.13: \( 5x^2 + 30x + 45 \)

\[
5(x+3)^2
\]

Find two real numbers \( b \), or one real number \( c \) such that the expressions is a perfect square trinomial.

5.5.21: \( x^2 + 6x + 81 \)

\[
\pm 18
\]

5.5.28: \( z^2 - 20z + c \)

\[
\text{\underline{100}}
\]

5.5.25: \( x^2 + 8x + c \)

\[
\text{\underline{16}}
\]

Factor the trinomial.

5.5.37: \( x^2 + 6x + 5 \)

\[
(x+1)(x+5)
\]

5.5.42: \( m^2 - 3m - 10 \)

\[
(m-5)(m+2)
\]

5.5.38: \( x^2 + 7x + 10 \)

\[
(x+2)(x+5)
\]

5.5.44: \( x^2 + 4x - 12 \)

\[
(x+6)(x-2)
\]

5.5.40: \( x^2 - 10x + 24 \)

\[
(x-4)(x-6)
\]

5.5.45: \( x^2 - 20x + 96 \)

\[
(x-8)(x-12)
\]

5.5.41: \( y^2 + 7y - 30 \)

\[
(y+10)(y-3)
\]

5.5.49: \( x^2 + 30xy + 216y^2 \)

\[
(x+12y)(x+18y)
\]

Factor the trinomial, if possible. (Note: Some of the trinomials may be prime.)
5.5.67: \(6x^2 - 5x - 25\) \quad 5.5.77: \(6b^2 + 19b - 7\)

\[ (3x+5)(2x-5) \quad (3b-1)(2b+7) \]

5.5.69: \(10y^2 - 7y - 12\) \quad 5.5.79: \(-2x^2 - x + 6\)

\[ (5y+4)(2y-3) \quad -(2x-3)(x+2) \]

5.5.70: \(6x^2 - x - 15\) \quad 5.5.85: \(4w^2 - 3w + 8\)

\[ (3x-5)(2x+3) \quad \text{Prime} \]

5.5.75: \(2t^2 - 7t - 4\) \quad 5.5.87: \(60y^3 + 35y^2 - 50y\)

\[ (2t+1)(t-4) \quad 5y(3y-2)(4y+5) \]

Factor the trinomial by grouping.

5.5.93: \(3x^2 + 10x + 8\) \quad 5.5.96: \(7x^2 - 13x - 2\)

\[ (3x+4)(x+2) \quad (7x+1)(x-2) \]

Factor the expression completely.

5.5.99: \(3x^3 - 3x\) \quad 5.5.102: \(16z^3 - 56z^2 + 49z\)

\[ 3x(x+1)(x-1) \quad 4(z-7)^2 \]

5.5.132: *Number Problem* Let \(n\) be an integer.

(a) Factor \(8n^3 + 12n^2 - 2n - 3\) so as to verify that it represents the product of three consecutive odd integers.

\[ (8n^3 + 12n^2 - 2n - 3) = (2n-1)(2n+1)(2n+3) \]

(b) If \(n = 15\), what are the three integers?

\[ 29, 31, 33; \]
6.1 Rational Expressions and Functions

Find the domain of the rational function.

6.1.1: \( f(x) = \frac{x^2 + 9}{4} \)
6.1.10: \( h(x) = \frac{4x}{x^2 + 16} \)
\((-\infty, \infty)\)
\((-\infty, \infty)\)

6.1.3: \( f(x) = \frac{4}{x - 3} \)
6.1.15: \( f(t) = \frac{5t}{t^2 - 16} \)
\((-\infty, 3) \cup (3, \infty)\)
\((-\infty, -4) \cup (-4, 4) \cup (4, \infty)\)

6.1.4: \( g(x) = \frac{-2}{x - 7} \)
6.1.17: \( g(y) = \frac{y + 5}{y^2 - 3y} \)
\((-\infty, 7) \cup (7, \infty)\)
\((-\infty, 0) \cup (0, 3) \cup (3, \infty)\)

Evaluate the rational function as indicated, and simplify. If not possible, state the reason.

6.1.23: \( f(x) = \frac{4x}{x + 3} \)
(a) \( f(1) = 1 \)
(b) \( f(-2) = -8 \)
(c) \( f(-3) \) undefined (division by 0)
(d) \( f(0) = 0 \)

6.1.27: \( h(s) = \frac{s^2}{s^2 - s - 2} \)
(a) \( h(10) = \frac{25}{22} \)
(b) \( h(0) = 0 \)
(c) \( h(-1) \) undefined
(d) \( h(2) \) undefined

Describe the domain.

6.1.30: Cost The cost \( C \) in millions of dollars for the government to seize \( p\% \) of an illegal drug as it enters the country is given by

\[
C = \frac{528p}{100 - p}.
\]
\([-\infty, 1) \cup (1, \infty)\)
6.1.31: \textit{Inventory Cost} The inventory cost \( I \) when \( x \) units of a product are ordered from a supplier is given by

\[
I = \frac{0.25x + 200}{x}.
\]

\[
\{2, 3, 4, \ldots\}
\]

Simplify the rational expression.

\[
6.1.43: \quad \frac{5x}{25} = \frac{x}{5}
\]

\[
6.1.45: \quad \frac{12x^2}{12x} = x, \quad x \neq 0
\]

\[
6.1.51: \quad \frac{x^2(x - 8)}{x(x - 8)} = x, \quad x \neq 0, x \neq 8
\]

\[
6.1.52: \quad \frac{a^2b(b - 3)}{b^3(b - 3)^2} = \frac{a^2}{b^2(b - 3)}
\]

\[
6.1.55: \quad \frac{y^2 - 49}{2y - 14} = \frac{y + 7}{2}, \quad y \neq 7
\]

\[
6.1.58: \quad \frac{u^2 - 12u + 36}{u - 6} = u - 6, \quad u \neq 6
\]

\[
6.1.60: \quad \frac{x^2 + 22x + 121}{3x + 33} = \frac{x + 11}{3}, \quad x \neq -11
\]

\[
6.1.65: \quad \frac{3x^2 - 7x - 20}{12 + x - x^2} = \frac{3(x + 4)}{12 + x - x^2}
\]

\[
6.1.66: \quad \frac{3x^2 - 7x - 20}{12 + x - x^2} = \frac{3}{x^2 - 3}
\]

\[
6.1.68: \quad \frac{x^2 + 22x + 121}{3x + 33} = \frac{x + 11}{3}, \quad x \neq -11
\]

\[
6.1.69: \quad \frac{3x^2 - 7x - 20}{12 + x - x^2} = \frac{3(x + 4)}{12 + x - x^2}
\]

\[
6.1.70: \quad \frac{3x^2 - 7x - 20}{12 + x - x^2} = \frac{3}{x^2 - 3}
\]

\[
6.1.71: \quad \frac{3xy^2}{xy^2 + x} = \frac{3y^2}{y^2 + 1}, \quad x \neq 0
\]

\[
6.1.72: \quad \frac{y^2 - 64x^2}{5(3y + 24x)} = \frac{y - 8x}{15}, y \neq -8x
\]

\[
6.1.73: \quad \frac{5xy + 3x^2y^2}{xy^3} = \frac{5 + 3xy}{y^2}, \quad x \neq 0
\]
6.1.78: \( \frac{x^2 + 4xy}{x^2 - 16y^2} \) 

\[ \frac{x}{x - 4y}, \quad x \neq 4y \]

6.1.87: *Average Cost* A machine shop has a setup cost of $2500 for the production of a new product. The cost of labor and material for producing each unit is $9.25.

(a) Write the total cost \( C \) as a function of \( x \), the number of units produced.

\[ C = 2500 + 9.25x \]

(b) Write the average cost per unit \( \bar{C} = C/x \) as a function of \( x \), the number of units produced.

\[ \bar{C} = \frac{C}{x} = \frac{2500 + 9.25x}{x} \]

(c) Determine the domain of the function in part (b).

\[ \{ 1, 2, 3, 4, \ldots \} \]

6.1.88: *Average Cost* A greeting card company has an initial investment of $60,000. The cost of producing one dozen card is $6.50.

(a) Write the total cost \( C \) as a function of \( x \), the number of dozens of cards produced.

\[ C = 60,000 + 6.50x \]

(b) Write the average cost per dozen \( \bar{C} = C/x \) as a function of \( x \), the number of dozens of cards produced.

\[ \bar{C} = \frac{60,000 + 6.50x}{x} \]

(c) Determine the domain of the function in part (b).

\[ \{ 1, 2, 3, 4 \} \]

(d) \( \notin 11.95 \)