ASSIGNMENT 4

DYLAN ZWICK’S MATH 1010 CLASS

2.5 Absolute Value Equations and Inequalities

Determine whether the value is a solution of the equation:

2.5.1: $|4x + 5| = 10, \ x = -3$  \hspace{1cm} 2.5.4: $\frac{1}{2}t + 4 = 8, \ t = 6$

Not a Solution  \hspace{1cm}  Not a Solution

2.5.2: $|2x - 16| = 10, \ x = 3$

Solution

Transform the absolute value equation into two linear equations:

2.5.5: $|x - 10| = 17$

$x - 10 = 17 \ \text{or} \ x - 10 = -17$

2.5.8: $|22k + 6| = 9$

$22k + 6 = 9 \ \text{or} \ 22k + 6 = -9$

2.5.7: $|4x + 1| = \frac{1}{2}$

$4x + 1 = \frac{1}{2} \ \text{or} \ 4x + 1 = -\frac{1}{2}$

Write the absolute value equations in standard form:

2.5.9: $|3x| + 7 = 8$  \hspace{1cm} 2.5.11: $3|2x| - 1 = 5$

$|3x| = 1$  \hspace{1cm}  |2x| = 2$

Date: Due Wednesday, September 23rd.
Solve the equations:

2.5.13: $|x| = 4$

$X = 4$ or $X = -4$

2.5.16: $|s| = 16$

$s = 16$ or $s = -16$

2.5.17: $|h| = 0$

$h = 0$

2.5.18: $|x| = -82$

No Solution

2.5.20: $\frac{1}{3}|x| = 2$

$X = 6$ or $X = -6$

2.5.21: $|x + 1| = 5$

$X = 4$ or $X = -6$

2.5.24: $\frac{7a + 6}{4} = 2$

$\frac{7a + 6}{4} = 2$ or $a = \frac{2}{7}$

2.5.26: $|3x - 2| = -5$

No Solution

2.5.27: $|5x - 3| + 8 = 22$

$X = \frac{17}{5}$ or $X = -\frac{11}{5}$

2.5.28: $|5 - 2x| + 10 = 6$

No Solution

2.5.30: $\frac{|x - 2|}{5} + 4 = 4$

$X = 2$

2.5.32: $4|5x + 1| = 24$

$|5x + 1| = 6$

$X = 1$ or $X = -\frac{7}{5}$

2.5.34: $2|4 - 3x| - 6 = -2$

$X = \frac{2}{3}$ or $X = 2$

2.5.35: $|x + 8| = |2x + 1|$

$X + 8 = 2X + 1$ or $X + 8 = -(2X + 1)$

$X = 7$ or $X = -3$

2.5.37: $|3x + 1| = |3x - 3|$

$3X + 1 = 3X - 3$ or $3X + 1 = -(3X - 3)$

No Solution

The only solution is $X = \frac{1}{3}$
2.5.40: \(3|2 - 3x| = |9x + 21|\)

\[3(2-3x) = 9x + 21\quad x = -\frac{5}{6}\]

The only solution is 0.

\[3(2-3x) = -(9x+21)\quad \text{No solution}\]

\[x = -\frac{5}{6}\]

Write an absolute value equation that represents the verbal statement:

2.5.42: The distance between \(-3\) and \(t\) is 5.

\[|-3 - t| = 5\]

Determine whether the \(x\)-value is a solution of the equality:

2.5.43: \(|x| < 3, x = 2\)

Solution

2.5.45: \(|x - 7| \geq 3, x = 9\)

Not a solution

Transform the absolute value inequality into a double inequality or two separate inequalities:

2.5.47: \(|y + 5| < 3\)

\[-3 < y + 5 < 3\]

2.5.50: \(|8 - x| > 25\)

\[8-x > 25\quad \text{or}\quad 8-x < -25\]

Solve the inequality:

2.5.51: \(|y| < 4\)

\[-4 < y < 4\]

2.5.53: \(|x| \geq 6\)

\[x \leq -6\quad \text{or}\quad x \geq 6\]

2.5.56: \(|4z| \leq 9\)

\[-\frac{9}{4} \leq z \leq \frac{9}{4}\]
2.5.65: \[ \frac{|y - 16|}{4} < 30 \]

\[ -104 < y < 136 \]

2.5.69: \[ \frac{|3x - 2|}{4} + 5 \geq 5 \]

\[ -\infty < x < \infty \]

2.5.67: \[ |0.2x - 3| < 4 \]

\[ -5 < x < 35 \]

2.5.81: Write an absolute value inequality that represents the interval:

\[ |x| \leq 2 \]

Write an absolute value inequality that represents the verbal statement:

2.5.85: The set of all real numbers \( x \) whose distance from 0 is less than 3.

\[ |x| < 3 \]

2.5.88: The set of all real numbers \( x \) for which the distance from 0 to 5 more than half of \( x \) is less than 13.

\[ \left| \frac{1}{2}x + 5 \right| < 13 \]
2.5.89: **Speed Skating** In the 2006 Winter Olympics, each skater in the 500-meter short track speed skating final had a time that satisfied the inequality \(|t - 42.238| \leq 0.412\), where \(t\) is the time in seconds. Sketch the graph of the solution of the inequality. What are the fastest and slowest times?

\[
|t - 42.238| \leq 0.412
\]

41.826 \(\leq t \leq 42.65\)

The fastest time is 41.826 seconds and the slowest time is 42.65 seconds.

2.5.91: **Accuracy of Measurements** In woodshop class, you must cut several pieces of wood to within \(\frac{3}{16}\) inch of the teacher’s specifications. Let \((s - x)\) represent the difference between the specification \(s\) and the measured length \(x\) of a cut piece.

(a) Write an absolute value inequality that describes the values of \(x\) that are within specifications.

\[
|s - x| \leq \frac{3}{16}
\]

(b) The length of one piece of wood is specified to be \(s = 5\frac{1}{8}\) inches. Describe the acceptable lengths for this piece.

\[
|5\frac{1}{8} - x| \leq \frac{3}{16}
\]

\[4\frac{15}{16} \leq x \leq 5\frac{5}{16}\]
3.1 The Rectangular Coordinate System

Plot the points on a rectangular coordinate system:

3.1.1: \((4, 3), (-5, 3), (3, -5)\)

3.1.3: \((-8, -2), (6, -2), (5, 0)\)

3.1.5: \((\frac{5}{2}, -2), (-2, \frac{1}{4}), (\frac{3}{2}, \frac{7}{2})\)

3.1.9: Determine the coordinates of the points:

\[\begin{align*}
A (0, -2, 4) \\
B (0, -2) \\
C (4, -2)
\end{align*}\]
3.1.12: Determine the coordinates of the points:

A \((-\frac{3}{2}, -2)\)
B \((5, 1)\)
C \((0, 4)\)

Plot the points and connect them with line segments to form the figure:

3.1.13: Square: \((2, 4), (5, 1), (2, -2), (-1, 1)\)

3.1.16: Triangle: \((-1, 3), (-2, -2), (3, 8)\)

3.1.20: Rhombus: \((-3, -3), (-2, -1), (-1, -2), (0, 0)\)
Find the coordinates of the point:

**3.1.21:** The point is located one unit to the right of the y-axis and four units above the x-axis.

\((1, 4)\)

**3.1.25:** The point is on the positive x-axis 10 units from the origin.

\((10, 0)\)

**3.1.27:** The coordinates of the point are equal, and the point is located in the third quadrant eight units to the left of the y-axis.

\((-8, 8)\)

Determine the quadrant in which the point is located without plotting it. \((x \text{ and } y \text{ are real numbers})\)

**3.1.29:** \((-3, -5)\)

\(\text{III}\)

**3.1.38:** \((x, -6)\)

\(\text{II} \quad \text{IV}\)

**3.1.33:** \((-9.5, -12.13)\)

\(\text{III}\)

**3.1.41:** \((x, y), xy > 0\)

\(\text{I} \quad \text{II}\)

Sketch a scatter plot of the points whose coordinates are shown in the table:

**3.1.43:** *Exam Scores* The table shows the time \(x\) in hours invested in studying for five different algebra exams and the resulting score \(y\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>5</th>
<th>2</th>
<th>3</th>
<th>6.5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>81</td>
<td>71</td>
<td>88</td>
<td>92</td>
<td>86</td>
</tr>
</tbody>
</table>
3.1.44: *Net Sales* The net sales $y$ (in billions of dollars) of Wal-Mart for the years 2003 through 2007 are shown in the table. The time in years is given by $x$. (Source: Wal-Mart 2007 Annual Report)

<table>
<thead>
<tr>
<th>$x$</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>226.5</td>
<td>252.8</td>
<td>281.5</td>
<td>308.9</td>
<td>345.0</td>
</tr>
</tbody>
</table>

3.1.46: *Fuel Efficiency* The table shows various speeds $x$ of a car in miles per hour and the corresponding approximate fuel efficiencies $y$ in miles per gallon.

<table>
<thead>
<tr>
<th>$x$</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>35</td>
<td>33.8</td>
<td>32.2</td>
<td>30</td>
<td>27.5</td>
</tr>
</tbody>
</table>
3.1.48: The figure is shifted to a new location in the plane. Find the coordinates of the vertices of the figure in its new location.

Complete the table of values. Then plot the solution points on a rectangular coordinate system.

3.1.49: \[
\begin{array}{c|cccc}
x & -2 & 0 & 2 & 4 & 6 \\
y = 5x + 3 & -7 & 3 & 13 & 23 & 33 \\
\end{array}
\]
3.1.50: \[
\begin{array}{cccccc}
x & -3 & 0 & 3 & 6 & 9 \\
\hline
y = 6x - 7 & -25 & 7 & 11 & 21 & 47
\end{array}
\]

Determine whether each ordered pair is a solution of the equation:

3.1.55: \(4y - 2x + 1 = 0\)

(a) \((0, 0)\) \[\text{Not a Solution}\]
(b) \((1, \frac{1}{2})\) \[\text{Solution}\]
(c) \((-3, \frac{7}{4})\) \[\text{Solution}\]
(d) \((1, \frac{3}{4})\) \[\text{Not a Solution}\]

3.1.60: \(y^2 - 4x = 8\)

(a) \((0, 6)\) \[\text{Not a Solution}\]
(b) \((-4, 2)\) \[\text{Not a Solution}\]
(c) \((-1, 3)\) \[\text{Not a Solution}\]
(d) \((7, 6)\) \[\text{Solution}\]
Plot the points and find the distance between them. State whether the points lie on a horizon or a vertical line.

3.1.61: \((3, -2), (3, 5)\)

3.1.66: \((\frac{3}{4}, 1), (\frac{3}{4}, -10)\)

3.1.69: \((1, 3), (5, 6)\)

3.1.74: \((0, -5), (2, -8)\)

\[\sqrt{(1-5)^2 + (3-6)^2} = 5\]

\[\sqrt{(0-2)^2 + (-5-(-8))^2} = \sqrt{13}\]

3.1.79: Show that the points are vertices of a right triangle.

Use the Distance Formula to determine whether the three points are collinear.

3.1.83: \((2, 3), (2, 6), (6, 3)\)

3.1.86: \((2, 4), (1, 1), (0, -2)\)

\(d_1 = \sqrt{13},\ d_2 = \sqrt{13},\ d_3 = \sqrt{26}\)

\((\sqrt{13})^2 + (\sqrt{13})^2 = (\sqrt{26})^2\) by the converse of the Pythagorean Theorem. It is a right angle.

Not collinear

Collinear
3.1.93: Numerical Interpretation For a handyman to install \( x \) windows in your home, the cost \( y \) is given by \( y = 150x + 425 \). Use \( x \)-values of 1, 2, 3, 4, and 5 to help describe the relationship between the number of windows \( x \) and the cost of installation \( y \).

\[
\begin{array}{c|ccccc}
 x & 1 & 2 & 3 & 4 & 5 \\
 y & 575 & 725 & 875 & 1025 & 1175 \\
\end{array}
\]

3.1.95: Football Pass A football quarterback throws a pass from the 10-yard line, 10 yards from the sideline. The pass is caught by a wide receiver on the 40-yard line, 35 yards from the same sideline, as shown in the figure. How long is the pass?

\[
\sqrt{(35-10)^2 + (40-10)^2} = \sqrt{1525} \approx 39.05
\]

The pass is about 39.05 yards long.
3.2 Graphs of Equations

Match the equation with the label of its graph.

A

B

C

D

E

F
3.2.1: \( y = 2 \)  
3.2.2: \( y = 2 + x \)  
3.2.3: \( y = 2 - x \)  
3.2.4: \( y = x^2 \)  
3.2.5: \( y = x^2 - 4 \)  
3.2.6: \( y = |x| \)  

Sketch the graph of the equations.

3.2.7: \( y = 3x \)  
3.2.9: \( y = 4 - x \)  
3.2.14: \( 2y + 5x = 6 \)  
\( y = -\frac{5}{2}x + 3 \)  
3.2.15: \( y = -x^2 \)  
3.2.18: \( y = 4 - x^2 \)  
3.2.19: \( -x^2 - 3x + y = 0 \)  
\( y = x^2 + 3x \)
3.2.20: \(-x^2 + x + y = 0\)
\[ y = x^2 - x \]

3.2.21: \(x^2 - 2x - y = 1\)
\[ y = x^2 - 2x - 1 \]

3.2.22: \(x^2 + 3x - y = 4\)
\[ y = x^2 + 3x - 4 \]

3.2.25: \(y = |x| + 3\)

3.2.27: \(y = |x + 3|\)

3.2.30: \(y = -x^3\)

Find the \(x\)-and \(y\)-intercepts (if any) of the graph of the equation.

3.2.31: \(y = 6x - 3\)
- \(x\)-intercept: \(\left(\frac{1}{2}, 0\right)\)
- \(y\)-intercept: \((0, -3)\)

3.2.34: \(y = \frac{3}{4}x + 15\)
- \(x\)-intercept: \((-20, 0)\)
- \(y\)-intercept: \((0, 15)\)

3.2.37: \(4x - y + 3 = 0\)
- \(x\)-intercept: \((-\frac{3}{4}, 0)\)
- \(y\)-intercept: \((0, 3)\)

3.2.40: \(y = |x| + 4\)
- \(x\)-intercept: none
- \(y\)-intercept: \((0, 4)\)

3.2.42: \(y = |x - 4|\)
- \(x\)-intercept: \((4, 0)\)
- \(y\)-intercept: \((0, 4)\)
Sketch the graph of the equation and show the coordinates of three solution points (including x- and y-intercepts).

3.2.57: \( y = 3 - x \)

3.2.60: \( y = -4x + 8 \)

3.2.62: \( y - 2x = -4 \)

3.2.65: \( 3x + 4y = 12 \)

3.2.69: \( 5x - y = 10 \)

3.2.72: \( y = x^2 - 16 \)

3.2.76: \( y = \frac{1}{x^2} \)

3.2.80: \( y = x(x + 2) \)
3.2.86: \( y = |x| + 4 \)

3.2.92: \( y = |x| + |x - 2| \)

3.2.93: **Straight-Line Depreciation** A manufacturing plant purchases a new molding machine for $230,000. The depreciated value \( y \) after \( t \) years is given by

\[
y = 230,000 - 25,000t, \quad 0 \leq t \leq 8.
\]

Sketch a graph of this model.

3.2.95: **Straight-Line Depreciation** Your company purchases a new delivery van for $40,000. For tax purposes, the van will be depreciated over a seven-year period. At the end of the 7 years, the value of the van is expected to be $5000.

(a) Find an equation that relates the depreciated value of the van to the number of years since it was purchased.

\[
\text{annual depreciation} \quad \frac{40,000 - 5000}{7} = 5000
\]

So \( y = 40,000 - 5000t, \quad 0 \leq t \leq 7 \)

(b) Sketch the graph of the equation.
(c) What is the $y$-intercept of the graph and what does it represent?

$$y\text{-intercept : } (0, 40,000)$$

It represents the value of the delivery van when it was purchased.

3.2.97: Hooke’s Law The force $F$ (in pounds) required to stretch a spring $x$ inches from its natural length is given by

$$F = \frac{4}{3}x, \quad 0 \leq x \leq 12.$$  

(a) Use the model to complete the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

(b) Sketch a graph of the model.

(c) Determine the required change in $F$ if $x$ is doubled. Explain your reasoning.

$F$ doubles because $F$ is directly proportional to $x$.

3.2.100: Exploration Graph the equations $y = x^2 + 1$ and $y = -(x^2 + 1)$ on the same set of coordinate axes. Explain how the graph of an equation changes when the expression for $y$ is multiplied by $-1$. Justify your answer by giving additional examples.

When the expression for $y$ is multiplied by $-1$, the graph is reflected in the $x$-axis.

Additional Example: $y = x^2$ and $y = -x^2$