ASSIGNMENT 15

DYLAN ZWICK’S MATH 1010 CLASS

1. SECTION 9.4

In Exercises 1-24, use properties of logarithms to evaluate the expression without a calculator (If not possible, state the reason)

9.4.1: \( \log_{12}(12^3) \)

9.4.2: \( \log_3(81) \)

9.4.3: \( \log_4 \left( \frac{1}{16} \right)^3 \)

9.4.5: \( \log_5(\sqrt[3]{5}) \)

9.4.6: \( \ln(\sqrt{e}) \)

9.4.7: \( \ln(14^0) \)

9.4.10: \( \ln(e^7) \)

9.4.11: \( \log_4(8) + \log_4(2) \)
9.4.12: $\log_6(2) + \log_6(3)$

9.4.13: $\log_8(4) + \log_8(16)$

9.4.15: $\log_3(54) - \log_3(2)$

9.4.16: $\log_5(50) - \log_5(2)$

9.4.18: $\log_3(324) - \log_3(4)$

9.4.21: $\ln(e^8) + \ln(e^4)$

9.4.25: Use $\log_4(2) = 0.500$ to approximate $\log_4(8)$

In Exercises 33, 35, use $\ln(3) \approx 1.0986, \ln(5) \approx 1.6094$ to approximate the expression.

9.4.33: $\ln(9)$

9.4.35: $\ln \left( \frac{5}{3} \right)$

In Exercises 41, 45, use the properties of logarithms to verify the statement.

9.4.41: $-3 \log_4(2) = \log_4 \left( \frac{1}{8} \right)$
9.4.45: $-3 \ln \left( \frac{1}{7} \right) = \ln(56) - \ln(8)$

In Exercises 47-74, use the properties of logarithms to expand the expression.

9.4.47: $\log_3(11x)$

9.4.49: $\ln(3y)$

9.4.50: $\ln(5x)$

9.4.52: $\log_3(x^3)$

9.4.53: $\log_4(x^{-3})$

9.4.55: $\log_4(\sqrt{3x})$

9.4.57: $\log_2 \left( \frac{z}{17} \right)$

9.4.60: $\ln \left( \frac{\sqrt{x}}{x + 9} \right)$

9.4.63: $\log_4[x^6(x + 7)^2]$
9.4.67: \( \ln(\sqrt{x(x + 2)}) \)

9.4.71: \( \ln\left(\sqrt[3]{\frac{x^2}{x + 1}}\right) \)

9.4.74: \( \log_5\left(\frac{x^2 y^5}{z^7}\right) \)

In Exercises 75-100, use the properties of logarithms to condense the expression.

9.4.75: \( \log_{12}(x) - \log_{12}(3) \)

9.4.78: \( \log_5(2x) + \log_5(3y) \)

9.4.81: \( 4 \ln(b) \)

9.4.83: \( -2 \log_5(2x) \)

9.4.88: \( \ln(6) - 3 \ln(z) \)

9.4.90: \( 4 \ln(2) + 2 \ln(x) - \frac{1}{2} \ln(y) \)

9.4.93: \( 2[\ln(x) - \ln(x + 1)] \)
9.4.96: \(5 \log_3(x) + \log_3(x - 6)\)

9.4.100: \(2 \log_5(x + y) + 3 \log_5(w)\)

9.4.105: Simplify \(\log_5(\sqrt{50})\)

9.4.113: The relationship between the number of decibels \(B\) and the intensity of a sound \(I\) in watts per centimeter squared is given by

\[ B = 10 \log_{10} \frac{I}{10^{-16}}. \]

Use properties of logarithms to write the formula in simpler form, and determine the number of decibels of a thunderclap with an intensity of \(10^{-3}\) watt per centimeter squared.

2. Section 9.5

In Exercises 1, 3, determine whether each value of \(x\) is a solution of the equation.

9.5.1: \(3^{2x-5} = 27\), (a) \(x = 1\) (b) \(x = 4\)

9.5.3: \(e^{x+5} = 45\), (a) \(x = -5 + \ln 45\) (b) \(x \approx -2.1933\)

In Exercises 7-34, solve the equation.

9.5.7: \(7^x = 7^3\)
9.5.8: \(4^x = 4^{6}\)

9.5.10: \(e^{x+3} = e^{8}\)

9.5.13: \(6^{2x} = 36\)

9.5.15: \(3^{2-x} = 81\)

9.5.17: \(5^x = \frac{1}{125}\)

9.5.20: \(3^{x+2} = \frac{1}{27}\)

9.5.21: \(4^{x+3} = 32^x\)

9.5.22: \(9^{x-2} = 243^{x+1}\)

9.5.23: \(\ln(5x) = \ln(22)\)

9.5.26: \(\log_5(2x) = \log_5(36)\)

9.5.29: \(\log_2(x + 3) = \log_2(7)\)

9.5.30: \(\log_4(x - 8) = \log_4(-4)\)
9.5.31: \( \log_5(2x - 3) = \log_5(4x - 5) \)

9.5.35: Simplify \( \ln(e^{2x-1}) \)

In Exercises 39-118, solve the exponential or logarithmic equation. (Round your answer to two decimal places.)

9.5.39: \( 3^x = 91 \)

9.5.40: \( 4^x = 40 \)

9.5.42: \( 2^x = 3.6 \)

9.5.45: \( 7^{3y} = 126 \)

9.5.47: \( 3^{2-x} = 8 \)

9.5.50: \( 12^{x-1} = 324 \)

9.5.51: \( 4e^{-x} = 24 \)

9.5.53: \( \frac{1}{4}e^x = 5 \)

9.5.56: \( 4e^{-3x} = 6 \)
9.5.58: $32(1.5)^x = 640$

9.5.61: $1000^{0.12x} = 25,000$

9.5.63: $\frac{1}{5}4^{x+2} = 300$

9.5.67: $7 + e^{2-x} = 28$

9.5.70: $6 - 3e^{-x} = -15$

9.5.72: $10 + e^{4x} = 18$

9.5.73: $17 - e^{4} = 14$

9.5.83: $\log_{10}(x) = -1$

9.5.85: $\log_{3}(x) = 4.7$

9.5.87: $4 \log_{3}(x) = 28$

9.5.92: $\log_{3}(6x) = 4$

9.5.93: $\ln(2x) = \frac{1}{5}$
9.5.99: \( \frac{3}{4} \ln(x + 4) = -2 \)

9.5.105: \( \log_4(x) + \log_4(5) = 2 \)

9.5.109: \( \log_5(x + 3) - \log_5(x) = 1 \)

9.5.114: \( \log_6(x - 5) + \log_6(x) = 2 \)

9.5.118: \( \log_3(2x) + \log_3(x - 1) - \log_3(4) = 1 \)

9.5.123: A deposit of $10,000 is placed in a savings account for 2 years. The interest for the account is compounded continuously. At the end of 2 years, the balance in the account is $11,051.71. What is the annual interest rate for this account?

9.5.125: Solve the exponential equation \( 5000 = 2500e^{0.09t} \) for \( t \) to determine the number of years for an investment of $2500 to double in value when compounded continuously at the rate of 9%.
3. **Section 9.6**

In Exercises 1-6, find the annual interest rate.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Balance</th>
<th>Time</th>
<th>Compounding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 500</td>
<td>$1004.83</td>
<td>10 years</td>
<td>Monthly</td>
</tr>
<tr>
<td>$ 1000</td>
<td>$36,581.00</td>
<td>40 years</td>
<td>Daily</td>
</tr>
<tr>
<td>$ 750</td>
<td>$8267.38</td>
<td>30 years</td>
<td>Continuously</td>
</tr>
</tbody>
</table>

In Exercises 7-12, find the time for the investment to double.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Rate</th>
<th>Compounding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2500</td>
<td>7.5%</td>
<td>Monthly</td>
</tr>
<tr>
<td>$18,000</td>
<td>8%</td>
<td>Continuously</td>
</tr>
<tr>
<td>$600</td>
<td>9.75%</td>
<td>Continuously</td>
</tr>
</tbody>
</table>

In Exercises 13-18, determine the type of compounding. Solve the problem by trying the more common types of compounding.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Balance</th>
<th>Time</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 500</td>
<td>$8954.24</td>
<td>10 years</td>
<td>6%</td>
</tr>
<tr>
<td>$ 750</td>
<td>$1587.75</td>
<td>10 years</td>
<td>7.5%</td>
</tr>
<tr>
<td>$ 4000</td>
<td>$4788.76</td>
<td>2 years</td>
<td>9%</td>
</tr>
</tbody>
</table>

In Exercises, 19,24, find the effective yield.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Compounding</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>Continuously</td>
</tr>
<tr>
<td>9%</td>
<td>Quaterly</td>
</tr>
</tbody>
</table>

9.6.27: Is it necessary to know the principal $P$ to find the doubling time in Exercises 7-12? Explain.
In Exercises 30, 35, find the principal that must be deposited under the specified conditions to obtain the given balance.

<table>
<thead>
<tr>
<th>Balance</th>
<th>Rate</th>
<th>Time</th>
<th>Compounding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5000</td>
<td>8%</td>
<td>5 years</td>
<td>Continuously</td>
</tr>
<tr>
<td>$1000</td>
<td>5%</td>
<td>1 year</td>
<td>Daily</td>
</tr>
</tbody>
</table>

9.6.37: You make monthly deposits of 30 dollars in a saving account at an annual interest rate of 8%, compounded continuously. Find the balance \( A = \frac{P(e^{rt} - 1)}{e^{(\frac{r}{12})}} \) after 10 years.

9.6.57: The population of the world is modeled by the equation \( P = \frac{11.7}{1 + 1.21e^{-0.0269t}} \), where \( P \) is in billions and \( t = 0 \) represents 1990. Use the model to predict the population in 2025.