Organize Your Work!

Suppose you are working the following problem. A drain can handle 1000 gallons per minute of water. How many cubic meters can be drained in an hour?  

57.75 in$^3 = 1$ qt, 2.54 cm = 1 in

First you look at what conversion factors you are given and decide how to go about the conversion. For volume you must make the following conversions: $gal \rightarrow qt \rightarrow in^3 \rightarrow cm^3 \rightarrow m^3$. And for time: $min \rightarrow hr$. **Organize your work like this.**

$$1000 \frac{gal}{min} \times \frac{4 \text{ qt}}{1 \text{ gal}} \times \frac{57.75 \text{ in}^3}{1 \text{ qt}} \times \frac{2.54^3 \text{ cm}^3}{1^3 \text{ in}^3} \times \frac{1^3 \text{ m}^3}{100^3 \text{ cm}^3} \times \frac{60 \text{ min}}{1 \text{ hr}} \approx 227.12 \frac{m^3}{hr}$$

1. Assuming that your heart beats 70 times per minute, how many times does your heart beat in 6 days?

**ANSWER:** $\frac{70 \text{ times}}{\text{min}} \times 6 \text{ days} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} = 604,800 \text{ times}.$

2. Suppose that two current exchange rates are US $1 = 7.74$ Hong Kong dollars and 1 Austrian shilling = US $0.084$. What is the value of 1 (H.K.) Hong Kong dollar in shillings?

**ANSWER:** $\frac{7.74 \text{ H.K.}}{\$1} \times \frac{\$0.084}{1 \text{ Shilling}} = 0.6502 \frac{\text{H.K.}}{\text{Shilling}}$

3. Given that 1 meter(m.) = 100 centimeters(cm.), find the number of cubic centimeters in a cubic meters.

**ANSWER:** $1 \text{ m.}^3 = 100^3 \text{ cm.}^3$, Dividing by $\text{cm.}^3, 1 \text{ m.}^3 / \text{cm.}^3 = 100^3 = 10^6$. So there are millions of cubic centimeters in a cubic centimeters.

4. Suppose that you are planning a vacation which will include an automobile trip in France. By checking the Internet, you’ve found the following information: 1 gallon = 3.785 liters; $\$1 \text{ US} = 6.5 \text{ French Francs}$. If gas in France recently cost 5.2 francs per liter, what would the equivalent be in US dollars per gallon?

**ANSWER:** $\frac{5.2 \text{ fr}}{L} \times \frac{3.785 \text{ L}}{1 \text{ gal.}} \times \frac{\$1 \text{ US}}{6.5 \text{ fr}} = $(5.2 x 3.785)/6.5/gal. = $\frac{3.0280}{\text{gallon}}$

5. How many a square picometers are in a square decameter? : ($\text{pico} = 10^{-12}, \text{deca} = 10^1$ )
ANSWER: A square pico-meters = \((\text{picometers})^2 = (10^{-12} \text{ m.})^2 = 10^{-24} \text{ m}^2\).

A square decameters = \((\text{decameter})^2 = (10^1 \text{ m.})^2 = 10^2 \text{ m}^2\).

Since a square pico-meters is smaller scale than the square decameters, we divide it by the square decameters. So it is, \(\frac{10^{-24}}{10^2} \text{ m}^2 = 10^{26}\). There are \(10^{26}\) numbers of square picometers in a square decameters.

6. Convert 12 cubic meters to cubic feet and to cubic centimeters. \((1 \text{ m} = 3.8 \text{ ft})\)

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12 \text{ m}^3 = 3.8^3 \text{ ft}^3 = 54.8720 \text{ ft}^3.
\]

\[
12 \text{ m}^3 \times \frac{54.8720 \text{ ft}^3}{1 \text{ m}^3} = 658.4640 \text{ ft}^3.
\]

Since \(1 \text{ m.} = 100 \text{ cm}\), \(1 \text{ m}^3 = 100^3 \text{ cm}^3 = 10^6 \text{ cm}^3\).

\[
12 \text{ m}^3 \times \frac{10^6 \text{ cm}^3}{1 \text{ m}^3} = 12,000,000 \text{ cm}^3.
\]

7. Wally ran 5 miles in 34.7 minutes. Find his speed in meters per second. \((1 \text{ mi.} = 1.6093 \text{ km})\)

\[
\text{ANSWER: } \text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{5 \text{ mi.}}{34.7 \text{ min.}} \times \frac{1.6093 \text{ km}}{1 \text{ mi.}} \times \frac{1000 \text{ m.}}{1 \text{ km}} \times \frac{1 \text{ min.}}{60 \text{ sec.}} = 3.8648 \frac{\text{m.}}{\text{sec.}}.
\]

8. You have three 60 watt light bulbs in your room. You use your lights every day for 4 hours. The price of electricity is $0.10 per kilowatt-hour. Kilowatt-hour is a unit of energy \((\text{kW·hr})\). How much did it cost to use the lights every day for 30 days?

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\text{ANSWER: There are three } 60 \text{ watt light bulbs, so } 3 \times 60 \text{ watt. When you use 4 hours for 30 days, it is } 4 \times 30 \text{ hr}, \text{total energy for using the light bulbs is } 3 \times 60 \text{ watt} \times 4 \times 30 \text{ hr} = 21600 \text{ watt·hr} = 21.6 \text{ kW·hr} \text{ So the total cost for those is } 21.6 \text{ kW} \times 10 \times \frac{\text{cents}}{\text{kW·hr}} = $2.16.( \text{Two dollars and sixteen cents})
\]

9. In 1999 the number of aerospace jobs in California was 153,000 down approximately 65% from the number in 1989. How many aerospace jobs were there in California in 1989?

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\text{ANSWER: } 153,000(\text{current value in 1999}), \text{which has been down 65% from the value “P”(unknown) in 1989. So we can write an equation (or relation) as}
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153,000 = P - 0.65P = (1 - 0.65)P = 0.35P
\]
10. The number of cars in Small town is currently 13,421 compared to 2,456 in 1965. Find the absolute and relative change in the percentage.

**ANSWER:**
Absolute change = current - old = 13,421 - 2,456 = 10,965
Relative Change = \( \frac{\text{current-old}}{\text{old}} \) = \( \frac{10,965}{2,456} \approx 4.46 \). It is 446% increase!

11. Given that a U.S. dollar is worth 1.36 Canadian dollars, how much larger is a U.S. dollar than a Canadian dollar? Answer in percentage terms.

**ANSWER:**
US dollar = 1.36 Canadian Dollars = (1 + 0.36) Canadian Dollars = (100% + 36%) Canadian Dollars. In the percentage sense, US dollar is 36% larger than Canadian dollar.

12. If Carol earns 50% more than William, how much, in percentage terms, less does William earn than Carol?

**ANSWER:** Let \( C \) be Carol’s earning, and let \( W \) be William’s earning. Carol earns 50% more than William, which can be described as \( C = W + 0.5W = (1 + 0.5)W = 1.5W \). If we solve for \( W \), then we have \( W = \frac{1}{1.5}C = \frac{2}{3}C = (1 - \frac{1}{3})C = C - \frac{1}{3}C \).
This says that William earns \( \frac{1}{3}(33.3\%) \) less than the Carol.

13. In a new TIME/CNN poll, 80% of working people say they’re not worried about losing their jobs, up 69% from last winter. What percentage of people said they were not worried last winter?

**ANSWER:** From the report, we see that 80% is current value. The old value of last winter is not know, we need to find it. Let it be “P%”. There was 69% increase from the old “P%”. We can express this as \( 80\% = P\% + 69\% \times P\% \). We can cancel the \% on both sides, \( 80 = P + 0.69P \).
Solve for \( P \), then \( P \) is \( \frac{80}{1.69} = 47.33 \). Last winter 47.33% of working people were not worried about losing their jobs.

14. Your dinner bill is $18.75. You leave $22. What percent tip did you leave? (Ignore taxes in this question.)
ANSWER: Let $T$ be tip. We can write the relation as $18.75 + T = 22$, so $T = 3.25$. In the relative sense, $3.25$ is 17.33% of $18.75$ from the calculation $3.25/18.75 = 0.1733$.

15. A high school reports that its students’ SAT scores were down by 20% for one year. The next year, however, they rose by 30%. The high school principal announces, “Overall, test scores have improved by 10% over the past two years.” Determine whether the concluding statement is true or false. If it is false, state the true change. Explain clearly.

ANSWER: Let $P$ be the SAT score in two years ago. There was 20% down one year ago, $P - 0.2P = 0.8P$. This score $0.8P$ was up by 30% on this year, it is $0.8P + 0.3 \times 0.8P = 0.8P + 0.24P = 1.04P = (1 + 0.04)P$, which is overall only 4% improvement over the past two years. Thus the concluding statement is False.

16. A newspaper reporter is writing an article on the change in the median selling price of houses in a certain area between 1950 and 1990. The reporter will summarize the data with one of the following statements:

(a) “The median selling price in 1990 is 275% more than what it was in 1950.”
(b) “The median selling price in 1990 is 275% of that price in 1950.”

Do the statements have the same meaning? If you say yes, explain why. If you say no, replace the underlined number in statement (b) with an appropriate number so that both statements will have the same meaning.

ANSWER: No. Let $C$ be the median selling price in 1990, and let $D$ be the median selling price in 1950. The statement (a) can be expresses as $C = D + 2.75D$. It is $C = (1 + 2.75)D = 3.75D$. Thus it says that $C$ is 375% of $D$. That is, the median selling price in 1990 is 375% of that price in 1950.

17. Explain about the usage of percentage points and percent.

ANSWER: We use the percentage points when it means of “absolute” value, and we use percent when it means of “relative” value.