1. (12 pts) Write the form of partial fraction decomposition of the rational expression. (Find all constants.)

\[
\frac{x - 3}{x^2 - x - 2}
\]

Answer: Write down the partial decomposition. It is

\[
\frac{x - 3}{x^2 - x - 2} = \frac{A}{x - 2} + \frac{B}{x + 1}
\]

Canceling out the denominator, the equation becomes \(x - 3 = A(x + 1) + B(x - 2)\). Plugging \(x = -1\), we obtain \(B = 4/3\). Plugging \(x = 2\), we have \(A = -1/3\). Thus the form is

\[
\frac{x - 3}{x^2 - x - 2} = \frac{-1}{3(x - 2)} + \frac{4}{3(x + 1)}
\]

2. Given a function

\[
f(x) = \frac{2 + x}{1 - x}
\]

a) (2 pts) Find the domain of the function.

Answer: All real numbers except \(x = 1\).

b) (6 pts) Find the horizontal and vertical asymptotes if there is any.

Answer: Horizontal Asymptote: As \(x \to -\infty\), \(y = -1\). Vertical Asymptote: Where the function is NOT defined. So \(x = 1\).

c) (8 pts) Sketch the graph of the function.

3. (12 pts) Find all real solutions of the polynomial equation.

\[x^3 + 2x^2 - 3x - 6 = 0\]

Answer: Among the factors of the number 6, \(x = -2\) is a solution by substitution. And then use the synthetic division, we find the equation

\[(x + 2)(x^2 - 3) = 0\]

Thus the solutions are \(x = -2, \pm \sqrt{3}\).

4. (12 pts) Find the inverse function to the following function.

\[f(x) = x^2 + 2, \quad x \geq 0\]

Answer: Switch \(x \) and \(y\), it becomes \(x = y^2 + 2\). Solve the equation for \(y\). Then \(y = \sqrt{x - 2}\).
5. Let a function \( f(x) = x^2 \). Sketch the each graph.

a) (6 pts) \( y = -f(-x) \) Answer: \( y = -f(-x) = -(-x)^2 = -x^2 \). It is the reflection by x-axis.

b) (6 pts) \( y = f(x + 3) \) Answer: It is a translation to the left in unit 3.

6. (12 pts) Write the slope-intercept forms of the equations of the lines through the given points perpendicular to the given line.

\[(2, 1) \text { point, } 4x - 2y = 3 \text { line} \]

Answer: The slope of the line \( 4x - 2y = 3 \) is 2. So the slope of the perpendicular line should be \(-1/2\). Then the desired line equation is \( y = -1/2x + b \). Plugging the point (2,1), it is \( 1 = -1/2(2) + b \). So \( b = 2 \). The equation is \( y = -1/2x + 2 \).

7. (12 pts) Solve the inequality.

\[
\frac{5}{x - 1} - \frac{2x}{x + 1} < 2
\]

Answer: Make a common denominator, the inequality becomes

\[
\frac{2x^2 - 3x - 9}{(x - 1)(x + 1)} > 0
\]

Factoring the numerator, it is \((x - 3)(x + 3/2)\). So the inequality is

\[(x - 1)(x + 1)(x - 3)(x + 3/2) > 0\]

The solutions are \( x < -3/2, -1 < x < 1, x > 3 \).

8. (12 pts) Find the domain of the composite function, \( g \circ f \).

\[ f(x) = \sqrt{x + 1}, \quad g(x) = x^2 \]

Answer: \( g \circ f(x) = g(\sqrt{x + 1}) = x + 1 \). The domain of the function \( f(x) \) is \( x \geq -1 \), which must be the domain of the composite function, too.