Final Prep: Early material

1. What is an arithmetic sequence?
   A sequence defined by the rule \( a_{n+1} = a_n + d \)
   for some \( d \in \mathbb{R} \).

2. What is a geometric sequence?
   A sequence defined by the rule \( a_{n+1} = ra_n \).

3. What’s the 170\(^{th} \) term of 3, 5, 7, 9, ...
   \[ 3 + 169(2) = 3 + 338 = 341 \]

4. What’s the 214\(^{th} \) term of -5, 15, -45, 135, ...
   \((-5)(-3)^{213}\)

5. For which kinds of sequences do you know how
   to find the sum of the first \( k \) terms?
   arithmetic sequences

6. For which kinds of sequences have we learned how
   to find the sum of all the terms?
   geometric sequences where \(-1 < r < 1\).

7. What’s the sum of the first 40 terms of -7, -4, -12, ...
   \[ \frac{40}{2} [a_1 + a_{40}] = 20 [-7 + (-7 + 39(3))] = 20 [-7 - 7 + 117] \]
   \[ = 20[103] = 2060 \]

8. What’s \( \sum_{i=1}^{\infty} \frac{7}{5^i} \)?
   \[ \frac{7}{5} = \frac{7 \cdot 5}{4 \cdot 5} = \frac{7}{4} \]
9. Find $\sum_{i=1}^{3} (1 - i^2)\\ \quad \quad (1-1^2) + (1-2^2) + (1-3^2) = 0 - 3 - 8 = -11$

10. Find $\sum_{i=1}^{30} 5\\ \quad \quad 5(30) = 150$

11. How many ways are there to order a set of 48 objects?\\ $48!$

12. How many ways can you choose and then order 17 objects from a set of 58 objects?\\ $\frac{58!}{(58-17)!} = \frac{58!}{41!}$

13. How many subsets of a set of 98 objects contain exactly 23 objects?\\ $\binom{98}{23}$

14. What does "options multiply" mean? To find the number of ways a sequence of decisions can be made, count how many ways each of those decisions can be made, and then multiply each of those numbers.

15. Write $\binom{7}{4}$ as a natural number in standard form.\\ $\frac{7!}{4!3!} = \frac{7\cdot6\cdot5\cdot4!}{4!\cdot6} = 7\cdot5 = 35$
16. \( f(x) = x^2 + 2 \), \( g(x) = 3x - 1 \). Find \( f \circ g(x) \) and \( g \circ f(x) \).

\[
\begin{align*}
\quad \; f \circ g(x) &= f(3x-1) = (3x-1)^2 + 2 \\
g \circ f(x) &= g(x^2+2) = 3(x^2+2) - 1
\end{align*}
\]

17. \( f(x) = 2(x-4)^3 + 1 \). Find \( f^{-1}(y) \).

\[
\begin{align*}
y &= 2(x-4)^3 + 1 \\
y - 1 &= 2(x-4)^3 \\
\frac{y - 1}{2} &= (x-4)^3 \\
\frac{\sqrt[3]{y - 1}}{2} &= x - 4
\end{align*}
\]

\[
\begin{align*}
x &= \sqrt[3]{\frac{y - 1}{2}} + 4 \\
f^{-1}(y) &= \sqrt[3]{\frac{y - 1}{2}} + 4
\end{align*}
\]

18. What are the implied domains of the following functions:

- \( f(x) = x \) \quad \text{R}
- \( f(x) = x^2 \) \quad \text{R}
- \( f(x) = x^3 \) \quad \text{R}
- \( f(x) = 4 \) \quad \text{R}
- \( f(x) = \sqrt[3]{x} \) \quad \text{[0, \infty)}
- \( f(x) = \sqrt[3]{x} \) \quad \text{R}
- \( f(x) = \frac{1}{x} \) \quad \text{R - {0}}
- \( f(x) = e^x \) \quad \text{R}
- \( f(x) = \log_e(x) \) \quad \text{(0, \infty)}
19. What’s the implied domain of \( f(x) = \frac{27}{3} x^5 - 3x^2 + 27 \)?

   \( \mathbb{R} \)

20. What’s the implied domain of \( r(x) = \frac{3x - 7}{x^2 - 4} \) ?

   \( x^2 - 4 \neq 0 \)
   \( x^2 \neq 4 \)
   \( x \neq 2 \) or \( -2 \)

   \( \mathbb{R} - \{ -2, 2 \} \)

21. What’s the implied domain of \( g(x) = \frac{1}{e^x} \) ?

   \( \mathbb{R} \)

22. What’s the implied domain of \( \sqrt[3]{7 - x} \) ?

   \( 7 - x \geq 0 \)
   \( 7 \geq x \)

   \( (-\infty, 7] \)

23. What’s the implied domain of \( 5x^2 - \sqrt[3]{2x - 3} \) ?

   \( \mathbb{R} \)

24. What’s the implied domain of \( 2x - \log_e (3x + 4) \) ?

   \( 3x + 4 > 0 \)
   \( 3x > 4 \)
   \( x > -\frac{4}{3} \)

   \( \left( -\frac{4}{3}, \infty \right) \)

25. What’s the implied domain of \( e^{\frac{3\sqrt{5x - 2}}{2}} + 3x^2 - 5 \) ?

   \( \mathbb{R} \)