Consider the following FALSE statements

(1) If I earn 20% less than you do, then you earn 20% more than I do.

(2) If the actual price of a car is 6.6% more than the labeled price (due to tax), then the labeled price is 6.6% less than the actual price.

We have discussed in class why the the first statement is false. (The second statement is false for the same reason.) One way to see this is to substitute actual numbers into the statement. Suppose that I make 20% less than you do and you make $10/hr. Then my wage is given by the formula

\[
\frac{\$10}{hr} - (0.2) \cdot \left(\frac{\$10}{hr}\right) = \frac{\$10}{hr} - \frac{\$2}{hr} = \frac{\$8}{hr}
\]

because my wage is found by taking your wage ($10/hr) and subtracting 20% of that (which is $2/hr.) On the other hand, 20% more than $8/hr is

\[
\frac{\$8}{hr} + (0.2) \cdot \left(\frac{\$8}{hr}\right) = \frac{\$8}{hr} - \frac{\$1.60}{hr} = \frac{\$9.60}{hr}.
\]

This difference is accounted for by the fact that a percentage increase (or decrease) depends on the starting value. 20% of the large number is more than 20% of the smaller number. This can be explained by the following fact:

\begin{center}
Percentage change is a relative change.
\end{center}

i.e., it depends on the starting value.

Quiz #3 problem 3 asked you to calculate the pre-tax cost of a car given the price after taxes. Fundamentally, this problem just statement (2) above. Several students calculated the pre-tax price of the car as if statement (2) were correct, but for the same reason as before, statement (2) is not true. So the calculation

\[
\$19,187.99 - (0.066) \cdot (\$19,187.99) = \$17,921.58
\]

is incorrect. We can check this: assuming the pre-tax cost was $17,921.58, the actual cost would be that amount plus tax, which is

\[
\$17,921.58 + (0.066)\$17,921.58 = \$19,104.40 \neq \$19,187.99.
\]

A good mental exercise would be to try and explain why the number we arrived at ($19,104.40) using the wrong method is smaller than the given number ($19,187.99.) Hint: consider what the starting values are and try to put the problem into the context of the boxed fact above.

By the way, the correct solution to problem 3 can be found with the rest of the quiz solutions on our webpage.