7 questions correctly, while your 90% score on the final means you answered 90 questions correctly. Thus, on the two exams combined, you answered 97 out of 110 questions correctly, which is 88.2% (because 97/110 = 0.882). This is much higher than the 80% "average" of the two individual exam percentages.

This example carries a very important lesson: As a general rule, you should never average percentages.

**EXAMPLE 15** Batting Average

In baseball, a player’s batting average represents the percentage of at-bats in which he got a hit. For example, a batting average of .350 means the player got a hit 35% of the times he batted. Suppose a player had a batting average of .200 during the first half of the season and .400 during the second half of the season. Can we conclude that his batting average for the entire season was .300 (the average of .200 and .400)? Why or why not? Give an example that illustrates your reasoning.

**Solution**  No. For example, suppose he had 300 at-bats during the first half of the season and 200 at-bats during the second half, for a total of 500 at-bats. His first-half batting average of .200 means he got hits on 20% of his 300 at-bats, or $0.2 \times 300 = 60$ hits. His second-half batting average of .400 means he got hits on 40% of his 200 at-bats, or $0.4 \times 200 = 80$ hits. For the season, he got a total of $60 + 80 = 140$ hits in his 500 at-bats. Thus, his season batting average was $140/500 = 0.28$, or .280—not the .300 found by averaging his first-half and second-half batting percentages. (In fact, the only case in which his season average would be .300 is if he had precisely the same number of at-bats in each half of the season.)

**Exercises 3A**

**Review Questions**

1. Describe the three basic uses of percentages. Give a sample statement that uses percentages in each of the three ways.
2. Distinguish between absolute and relative change. Give an example that illustrates how we calculate a relative change.
3. Distinguish between absolute and relative difference. Give an example that illustrates how we calculate a relative difference.
4. Explain the difference between the key words of and more than when dealing with percentages. How are their meanings related?
5. Explain the difference between the terms percent (%) and percentage points. Give an example of how they can differ for the same situation.
6. Give an example to explain why, in general, it is not legitimate to average percentages.

**Does It Make Sense?**

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. In many European countries, the percentage change in population has been negative in recent decades.
8. The value of our home has tripled since we bought it—that’s a 200% increase in value!
9. My older child weighs 25% more than my younger child.
10. I’ve decreased my caloric intake 125%, which has helped me lose weight.
11. If you earn 20% more than I do, then I must earn 20% less than you do.
12. If they keep cutting taxes by 10% every year, in a decade we won’t be paying any tax at all!
13. We found that these rare cancers were 700% more common in children living near the toxic landfill than in the general population.
14. I have a 60% average on our assignments going into the final exam, but I still hope to raise my course average to 70% by getting an 80% on the final. (The final is worth 25% of the final grade.)
15. The rate of return on our fund increased by 50%, to 15%.
16. My bank increased the interest rate on my savings account 100%, from 2% to 4%, so my bank balance is now doubling every month.

Basic Skills & Concepts

Review of Fractions. Exercises 17–32 use skills covered in the Brief Review on p. 135. For each exercise, express the given number in three forms: fraction, decimal, and percentage.

17. \( \frac{1}{4} \) 18. 120%
19. 0.45 20. 0.652
21. 0.33333... 22. 2.34
23. 23% 24. 0.98
25. \( \frac{1}{4} \) 26. 45%
27. 1.34 28. 1.01
29. 0.65 30. \( \frac{2}{3} \)
31. 99% 32. 125%

Review of Ratios. Exercises 33–44 use skills covered in the Brief Review on p. 140. Each exercise gives a pair of quantities, \( A \) and \( B \). Compare them in three ways. First, find the ratio of \( A \) to \( B \). Then, find the ratio of \( B \) to \( A \). Finally, complete this statement: \( A \) is _____ percent of \( B \).

33. \( A = 10 \) and \( B = 2 \) 34. \( A = 150 \) and \( B = 25 \)
35. \( A = 550 \) and \( B = 50 \) 36. \( A = 45 \) and \( B = 550 \)
37. \( A = 75 \) and \( B = 480 \) 38. \( A = 33 \) and \( B = 123 \)
39. \( A \) is the 7.5 million population of North Carolina and \( B \) is the 16.5 million population of Texas.
40. \( A \) is the 2.8 million population of Chicago and \( B \) is the 0.75 million population of Baltimore.
41. \( A \) is the 359,000 births recorded in the United States in a recent July and \( B \) is the 290,000 births recorded in the United States in a recent January.
42. \( A \) is the average U.S. household size of 3.1 people in 1970 and \( B \) is the average U.S. household size of 2.6 people in 2003.
43. \( A \) is the average size of a new single-family home in 1970, which was 1500 square feet. \( B \) is the average size of such a new home in 2003, which was 2200 square feet.
44. \( A \) is the median price of $23,000 for a single-family house in the United States in 1970 and \( B \) is the median price of $163,000 for a single-family house in the United States in 2003.

Percentages as Fractions. Each statement in Exercises 45–50 gives two numbers. Express the first number as a percentage of the second.

45. 25 women in a room of 113 people
46. 345 blooming tulips in a field of 398 tulips
47. 1234 people who voted for the losing candidate out of 3009 voters
48. 38 men at a convention of 236 people
49. 23 purple jelly beans in a jar of 123 jelly beans
50. 2345 people who voted for the winning candidate out of 4023 voters

51. Salary Comparison. John's salary grew from $20,000 in 2000 to $28,000 in 2003. Mary's salary grew from $25,000 to $35,000 in the same period. Whose salary grew more in absolute terms? In relative terms? Explain.

52. Population Comparison. The population of Clarkville grew from 1400 to 2200 in the past decade, while the population of Centerville grew from 27,000 to 29,000. Which town had the greater absolute growth in population? Which town had the greater relative growth? Explain.

Percentage Change. Exercises 53–56 each describe measurements made at two different times. Find the absolute and relative change (as a percentage) in each case.

53. The population of the United States grew from 76 million in 1900 to 281 million in 2000.
54. Consumer debt in the United States increased from $130 billion in 1974 to $1.75 trillion ($1750 billion) in 2003.
55. The number of daily newspapers in the United States was 2226 in 1900 and 1483 in 2000.
56. The annual number of deaths from cardiovascular disease in the United States decreased from 1,088,000 in 1970 to 950,000 in 2000.
Percentage Difference. Exercises 57–62 each state two measurements. Find the absolute and relative difference (as a percentage) in each case. Assume that the first quantity is the compared value and the second quantity is the reference value.

35. The daily circulation of the Wall Street Journal is about 1.77 million (the largest in the country). The daily circulation of the New York Times is about 1.07 million (the third largest in the country).
36. In a recent year, Ford sold 401,049 Tauruses and Honda sold 382,296 Accords.
37. In a recent year, France ranked as the number one tourist destination, with about 66 million international arrivals. The United States ranked second, with about 48 million international arrivals.

65. The population of Montana is 20% less than the population of New Hampshire, so Montana's population is ___% of New Hampshire's population.
66. Henry earns 45% less than Ingrid, so his salary is ___% of Ingrid's salary.

Prices and Sales. Fill in the blanks in Exercises 67–70.
67. The wholesale cost of a car is 50% less than its retail cost. Therefore, the retail cost is ___ times the wholesale cost.
68. A store is having a 15% off sale. Therefore, a sale price is ___ times an item's original price.
69. The retail cost of a TV is 40% more than its wholesale cost. Therefore, the retail cost is ___ times the wholesale cost.
70. A store is having a 40% off sale. The sale price for an item with a regular cost of $80 is ___.

Percentages of Percentages. Exercises 71–74 describe changes in which the measurements themselves are percentages. Express each change in two ways: (1) as an absolute difference in terms of percentage points and (2) as a relative difference in terms of a percentage.
71. In a 22-year period, the percentage of high school seniors using alcohol decreased from 68.2% to 52.7%.
72. In a 25-year period, the percentage of the world's population living in developed countries decreased from 27% to 20%.
73. In the past four decades, the five-year survival rate for all forms of cancer increased from 39% to 60%.
74. The percentage of all bachelor's degrees awarded to women rose from 44% in 1972 to 58% in 2000.

Care in Wording. Assume that 30% of city employees in Carson City ride the bus to work. Consider the following two statements:
- The percentage of city employees who ride the bus to work is 10% higher in Freetown than in Carson City.
- The percentage of city employees who ride the bus to work is 10 percentage points higher in Freetown than in Carson City.

For each case, state the percentage of city employees in Freetown who ride the bus to work. Briefly explain why the two statements have different meanings.

75. Ambiguous News. The average annual precipitation on Mt. Washington, New Hampshire, is 90 inches. During
one particularly wet year, different news reports carried the following statements.

- The precipitation this year is 200% of normal.
- The precipitation this year is 200% above normal.

What does each of these statements imply about the precipitation during this year? Do the two statements have the same meaning? Explain.

Solving Percentage Problems. Solve the percentage problems in Exercises 77–84.

77. You purchase a bicycle with a labeled (pre-tax) price of $699. The local sales tax rate is 7.6%. What is the final cost?

78. The final cost of your new shoes is $107.69. The local sales tax rate is 6.2%. What was the labeled (pre-tax) price?

79. You purchase a car with a labeled (pre-tax) price of $17,600. The local sales tax rate is 6.5%. What is the final cost?

80. The final cost of your new HDTV is $3706.30. The local sales tax rate is 8.1%. What was the labeled (pre-tax) price?

81. Your dinner bill is $18.75. You leave $2.2. What percent tip did you leave? (Ignore taxes in this question.)

82. Your dinner bill is $42.50. You leave $5.0. What percent tip did you leave? (Ignore taxes in this question.)

83. Consider the statement “The smoking rate among tenth-graders jumped 45 percent, to 18.3 percent.” What percentage of tenth-graders smoked before the increase?

84. Consider the statement: “The smoking rate among twelfth-graders jumped 20 percent, to 22 percent.” What percentage of twelfth-graders smoked before the increase?

Shifting Reference Value. In each of Exercises 85–88, determine whether the concluding statement is true or false. If it is false, state the true change. Explain clearly.

85. Sue went on a weight loss program and lost 10% of her weight. However, she then gained 10% (from her lowest weight). In the end, her weight was unchanged.

86. You receive a pay raise of 5%, then receive a pay cut of 5%. After the two changes in pay, your salary is unchanged.

87. Your property taxes increase 5% in one year and increase 6% the next year. The total increase in taxes over the two years is 11%.

88. A high school reports that its students’ SAT scores were down by 20% for one year. The next year, however, they rose by 30%. The high school principal announces, “Overall, test scores have improved by 10% over the past two years.”

Is It Possible? Decide whether each claim in Exercises 89–94 is possible. If it is, explain how. If not, explain why not.

89. The price of cellular service has decreased 150% in the last ten years.

90. Mary is 100% shorter than Vivian.

91. The price of real estate in this city has increased 450% in the last ten years.

92. Super Oats have 100% more calcium than Regular Oats.

93. Your computer is 250% faster than mine.

94. Your computer is 250% slower than mine.

95. Average Percentages. You are a teacher. Your first-period class, with 25 students, had a mean score of 86% on the midterm exam. Your second-period class, with 30 students, had a mean score of 84% on the same exam. Does it follow that the mean score for both classes is 85%? Explain.

96. Average Percentages. A player has a batting average over many games of .400. In his next game, he goes 2 for 4, which is a batting average of .500 for the game. Does it follow that his new batting average is \((.400 + .500)/2 = .450\)? Explain.

Further Applications

Analyzing Percentage Statements. For Exercises 97–100, decide if the statement is true or false. Explain.

97. The class is 60% women and 10% of the women have blonde hair, so blonde-haired women comprise \(60\% \times 10\% = 6\%\) of the class.
3A Uses and Abuses of Percentages

108. "Factory-to-dealer sales of mobile video and navigation devices amounted to more than $450 million last year, a 54% jump over the previous year."  
Question: What were the sales of mobile video and navigation devices in the previous year?

109. "Oracle's stock rose $13.48 at the end of the week, up 3% over the previous week's close."  
Question: What was the price of Oracle's stock at the previous week's close?

110. "The average cost of bringing a barrel of oil out of the ground in the U.S. is $10. In Saudi Arabia, it's about $2.50. And in Iraq, it's less than $1."  
Questions: How do the costs of bringing a barrel of oil out of the ground in Iraq and in the United States compare, in terms of percentages? How do the costs of bringing a barrel of oil out of the ground in Saudi Arabia and in the United States compare, in terms of percentages?

Web Project

Find useful links for Web Projects on the text Web site: www.aw.com/bennett-briggs

111. Web News Percentages. Go to the Web site for a newspaper and look for at least three stories with percentages. Explain the percentages in each case.

In the News

112. Percentages. Find three recent news reports that quote percentages. In each case, describe the use of the percentage (as a fraction, to describe change, or for comparison) and explain its context.

113. Percentage Change. Find a recent news report that quotes a percentage change. Describe the meaning of the change.

114. Abuse of Percentages. Find a news article or report in which the use of a percentage is either suspicious or wrong. If possible, clarify or correct the statement.
intelligent species would eventually emerge on Earth? We divide the Sun's remaining 5 billion years by the 50 million years between successive intelligent species:

\[
\frac{5 \text{ billion years}}{50 \text{ million years}} = \frac{5 \times 10^9 \text{ years}}{5 \times 10^7 \text{ years}} = 100
\]

Even at 50 million years per shot, the Earth would have time for some 100 more intelligent species to emerge. Perhaps one of those species will not destroy itself, and descendants of the Earth might move on to other star systems by the time the Sun finally dies. Perhaps that species will be ours.

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**EXERCISES 3B**

**Review Questions**

1. Briefly describe scientific notation. How is it useful for writing large and small numbers? How is it useful for making approximations?

2. Explain how we can use estimation to put numbers in perspective. Give an example.

3. What is an order of magnitude estimate? Explain why such an estimate can be useful even though it may be as much as 10 times too large or too small.

4. Explain how we can use comparisons to put numbers in perspective. Give an example.

5. Describe three common ways of expressing the scale of a map or model. How would you show a scale of 1 cm = 100 km graphically? How would you describe it as a ratio?

6. Explain how we can use scaling to put numbers in perspective. Give an example.

7. Suppose that the Sun were the size of a grapefruit. How big and how far away would the Earth be on this scale? How far would the nearest stars (besides the Sun) be?

8. Describe several ways of putting each of the following in perspective: the size of a large university; $1\text{ billion}$; the size of an atom; the Sun's remaining lifetime.

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**Does It Make Sense?**

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

9. I read a book that had $10^5$ words in it.

10. I've seen about $10^{30}$ commercials on TV.

11. I live in an apartment building that is 200 feet tall.

12. In total, Americans spend about a billion dollars per year on housing costs (rent and home mortgage payments).

13. During a recent sold-out NFL football game, the star player signed autographs for every single person in attendance.

14. The CEO of the company earned more money last year than the company's 500 lowest paid employees combined.

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**Basic Skills & Concepts**


15. Convert each of the following numbers from scientific notation and write its name.

   *Example*: $2 \times 10^3 = 2000 = \text{two thousand}

   a. $5 \times 10^6$

   b. $7 \times 10^9$

   c. $2 \times 10^{-2}$

   d. $8 \times 10^{11}$

   e. $1 \times 10^{-1}$

   f. $9 \times 10^{-4}$

16. Convert each of the following numbers from scientific notation and write its name.

   a. $7.0 \times 10^5$

   b. $1.5 \times 10^9$

   c. $3.0906 \times 10^3$

   d. $2.3 \times 10^6$

   e. $2.2 \times 10^{-4}$

   f. $7.06 \times 10^{-3}$

17. Write each of the following numbers in scientific notation.

   a. 600

   b. 0.9

   c. 50,000

   d. 0.003

   e. 0.0005

   f. 70,000,000,000
Write each of the following numbers in scientific notation.

a. 64,598  
   b. 0.92
   c. 500,098  
   d. 0.002 \times 10^6
   e. 250 million  
   f. -23,800

Do the following operations without a calculator and show your work clearly. Be sure to express answers in scientific notation. You may round your answers to two decimal places (as in \(3.2 \times 10^2\)).

a. \((3 \times 10^4) \times (8 \times 10^2)\)
   b. \((6.3 \times 10^5) + (1.5 \times 10^4)\)
   c. \((9 \times 10^3) \times (5 \times 10^{-7})\)
   d. \((4.4 \times 10^{20}) - (2.0 \times 10^{11})\)

Do the following operations without a calculator and show your work clearly. Be sure to express answers in scientific notation. You may round your answers to two decimal places (as in \(3.2 \times 10^2\)).

a. \((8 \times 10^{12}) + (4 \times 10^6)\)
   b. \((7.5 \times 10^{21}) + (1.5 \times 10^{13})\)
   c. \((3.2 \times 10^{22}) + (1.6 \times 10^{-14})\)
   d. \((6 \times 10^{10}) - (5 \times 10^9)\)

**Don’t Look That Different!** In Exercises 21–22, compare each pair of numbers. By what factor do the numbers differ?

**Example:** \(10^8\) is \(10^6\), or 100, times as large as \(10^4\).

a. \(10^{26}, 10^{28}\)
   b. \(10^{17}, 10^{27}\)
   c. 1 billion, 1 million
   d. 7 trillion, 7 thousand
   e. \(2 \times 10^{-6}, 2 \times 10^{-8}\)
   f. \(6.1 \times 10^{22}, 6.1 \times 10^{29}\)
   g. 250 million, 5 billion
   h. \(9.3 \times 10^9, 3.1 \times 10^{-2}\)
   i. \(10^{-6}, 2 \times 10^{-13}\)
   j. 1 thousand, 1 thousandth
   k. \(10^{12}, 10^{-9}\)

Using Scientific Notation. Rewrite each statement in Exercises 23–28 with a number in scientific notation.

a. Total annual energy consumption in the United States is about 100,000,000,000,000,000,000,000 joules.
   b. The United States imports about 420 million gallons of crude oil each day.
   c. The hard drive on my computer has a capacity of 12.0 gigabytes. (Hint: Recall that the prefix *giga* means 1 billion.)

26. The area of the Earth’s surface is 509,600,000 square kilometers.
27. The diameter of a typical bacterium is about 0.000001 meter.
28. A beam of light can travel the length of a football field in about 30 nanoseconds. (Hint: Recall that *nano* means one billionth.)

**Approximation with Scientific Notation.** In Exercises 29–30, make an estimate of the answer without a calculator, showing your method of estimation. Then do the exact calculation (with a calculator if necessary), and describe how well your approximation technique worked.

29. a. \(20,000 \times 100\)
   b. \(9642 \div 31\)
   c. \(-12.5 \times 11,890\)
   d. \(250 \text{ million} \times 40\)
   e. \(7.453 \times 291\)
   f. \(6,570,999 \div 32.7\)
30. a. \(5.6 \text{ billion} \times 200\)
   b. \(4 \text{ trillion} \div 260 \text{ million}\)
   c. \(9,000 \times 54,986\)
   d. \(3 \text{ billion} \div 25,000\)
   e. \(59,873 \times 341\)
   f. \(43 \div 765\)

**Perspective Through Estimation.** Make estimates as needed to answer Exercises 31–34.

31. Which is bigger, the height of a 10-story apartment building or the length of a football field? By how much?
32. Could a person walk across the United States (New York to California) in a year? If not, about how long would it take?
33. Which is more, the number of miles Americans fly each year or the number of miles Americans drive each year?
34. Which holds more people, a football stadium or 10 movie theaters?

**Orders of Magnitude Estimates.** For each quantity described in Exercises 35–40, make an order of magnitude estimate. Be sure to explain your estimates clearly.

35. The total amount of money spent each year by Americans going to the movies
36. The number of breaths you take in a week
37. The amount of water you drink in a year
38. The total number of words in this textbook
39. The amount of gasoline an average adult uses per year
40. The total amount you spend on food in a year
Energy Comparisons. Use data from Table 3.1 in Exercises 41–46.

41. How many average candy bars would you have to eat to supply the energy needed for four hours of running?

42. How many liters of oil are required to supply the electrical energy needs of an average home for a month?

43. Compare the energy released by burning 1 kilogram of coal to that released by fission of 1 kilogram of uranium-235.

44. Compare the energy released by burning 1 liter of oil to that released by fusion of the hydrogen in 1 liter of water.

45. If you could generate energy by fusing the hydrogen in ordinary tap water, how much water would you need to generate the electrical energy used daily by a typical home?

46. If you could generate energy by fusing the hydrogen in ordinary tap water, how much water would you need to meet all the world’s current energy consumption?

47. How many kilograms of uranium would be required to supply the energy needs of the United States for one year using fission?

48. Suppose that we could somehow capture all the energy released by the Sun for just one second. Would this energy be enough to supply U.S. energy needs for a year? Explain.

Scale Ratio. Find the scale ratio for each of the maps described in Exercises 49–56.

49. 1 centimeter on the map represents 1 kilometer on the ground.

50. 1 inch on the map represents 1 mile on the ground.

51. 2 inches on the map represents 0.5 mile on the ground.

52. 3 inches on the map represents 2 miles on the ground.

53. 5 cm (map) = 100 km (actual)

54. 10 cm (map) = 10 km (actual)

55. 1 ft (map) = 100 m (actual)

56. 1 ft (map) = 1 km (actual)

57. Scale Model Solar System. The following table gives size and distance data for all the planets. Calculate the scaled size and distance for each planet using a 1 to 10 billion scale model solar system. Give your results in table form. Then write one or two paragraphs that describe your findings in words and give perspective to the size of our solar system.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Diameter (km)</th>
<th>Average distance from Sun (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>4,880</td>
<td>57.9 million</td>
</tr>
<tr>
<td>Venus</td>
<td>12,100</td>
<td>108.2 million</td>
</tr>
<tr>
<td>Earth</td>
<td>12,760</td>
<td>149.6 million</td>
</tr>
<tr>
<td>Mars</td>
<td>6,790</td>
<td>227.9 million</td>
</tr>
<tr>
<td>Jupiter</td>
<td>143,000</td>
<td>778.3 million</td>
</tr>
<tr>
<td>Saturn</td>
<td>120,000</td>
<td>1,427 million</td>
</tr>
<tr>
<td>Uranus</td>
<td>52,000</td>
<td>2,870 million</td>
</tr>
<tr>
<td>Neptune</td>
<td>48,400</td>
<td>4,497 million</td>
</tr>
<tr>
<td>Pluto</td>
<td>2,260</td>
<td>5,900 million</td>
</tr>
</tbody>
</table>

58. Interstellar Travel. The fastest spaceships launched to date are traveling away from Earth at speeds of about 50,000 kilometers per hour. How long would such a spaceship take to reach Alpha Centauri? (Hint: See Example 8.) Based on your answer, write one or two paragraphs discussing whether interstellar travel is a realistic possibility today.

59. Universal Timeline. According to modern astronomy, the universe is about 14 billion years old. Suppose you represent this age with a timeline 100 meters long. What distance along the timeline represents 1 billion years? Written human history extends back about 10,000 years. How far would that be on the timeline?

Further Applications

Making Numbers Understandable. Restate the facts in Exercises 60–67 as indicated.

60. There are approximately 4 million births in the United States in a year. Express this quantity in births per minute.

61. There are approximately 2.2 million marriages in the United States each year. Express this quantity in marriages per day.

62. There are approximately 132 million births worldwide in a year. Express this quantity in births per minute.

63. There are roughly 30,000 suicides in the United States each year. Express this quantity in suicides per day.
American businesses spend about $250 billion on advertising each year. Express this quantity as the height of a stack of $1 bills. (Hint: Measure the thickness of a stack of ten $1 bills.)

63. The national debt was about $6.3 trillion at the end of 2003. Express this quantity as the height of a stack of $1 bills.

64. Approximately 720,000 Americans die of a heart attack each year. Express this quantity in heart attacks per day.

65. Approximately 1.7 million college students receive bachelor’s degrees each year. Express this number of students in terms of the number of football stadiums (capacity 70,000) that could be filled by these graduates.

66. CO₂ Emissions. For every gallon of gasoline burned by an automobile, approximately 10.2 kilograms of carbon dioxide are emitted into the atmosphere. Estimate the total amount of carbon dioxide added to the atmosphere by all automobiles travel in the United States over the past year.

67. The Amazing Amazon. An issue of National Geographic contained the following statement:

Dropping less than two inches per mile after emerging from the Andes, the Amazon drains a sixth of the world’s runoff into the ocean. One day’s discharge at its mouth—4.5 trillion gallons—could supply all U.S. households for five months.

Based on this statement, determine how much water an average U.S. household uses each month. Does this answer seem reasonable? Explain any estimates you make.

68. Wood for Energy? A total of about 180,000 terawatts of solar power reaches the Earth’s surface, of which about 0.006% is used by plants for photosynthesis. Of the energy that goes to photosynthesis, about 1% ends up stored in plant matter (including wood). (Recall that 1 watt = 1 joule/s; 1 terawatt = 10¹² watts.)

a. Calculate the total amount of energy that becomes stored in plant matter each second.

b. Suppose that power stations generated electricity by burning plant matter. If all the energy stored in plants could be converted to electricity, what average power, in terawatts, would be possible? Would it be enough to meet world electricity demand, which is of order 10 terawatts?

c. Based on your answer to b, can you draw any conclusions about why humans depend on fossil fuels, such as oil and coal, which are the remains of plants that died long ago? Explain.

71. Stellar Corpses: White Dwarfs and Neutron Stars. A few billion years from now, after exhausting its nuclear engines, the Sun will become a type of remnant star called a white dwarf. It will still have nearly the same mass (about 2 x 10³⁰ kg) as the Sun today, but its radius will be only about that of the Earth (about 6400 km).

a. Calculate the average density of the white dwarf in units of kilograms per cubic centimeter.

b. What is the mass of a teaspoon of material from the white dwarf? (Hint: A teaspoon is about 4 cubic centimeters.) Compare this mass to the mass of something familiar (for example, a person, a car, a tank).

c. A neutron star is a type of stellar remnant compressed to even greater densities than a white dwarf. Suppose that a neutron star has a mass 1.4 times the mass of the Sun but a radius of only 10 kilometers. What is its density? Compare the mass of 1 cubic centimeter of neutron star material to the total mass of Mt. Everest (about 5 x 10¹⁵ kg).

Sampling Problems. Sampling techniques can be used to estimate physical quantities. To estimate a large quantity, you might measure a representative small sample and find the total quantity by “scaling up.” To estimate a small quantity, you might measure several of the small quantities together and “scale down.” Carry out the necessary measurements and use sampling techniques to answer Exercises 72–75.

Example: How thick is a sheet of paper?

Solution: One way to estimate the thickness of a sheet of paper is to measure the thickness of a ream (500 sheets) of paper. A particular ream was 7.5 centimeters thick. Thus, a sheet of paper from this ream was 7.5 cm / 500 = 0.015 cm thick, or 0.15 millimeter.

72. How much does a sheet of paper weigh?

73. How thick is a penny? a nickel? a dime? a quarter? Would you rather have your height stacked in pennies, nickels, dimes, or quarters? Explain.

74. How much does a grain of sand weigh? How many grains of sand are in a typical playground sand box?

75. How many stars are visible in the sky on the clearest, darkest nights? How could astronomers estimate the total number of stars in the universe?
Web Projects

Find useful links for Web Projects on the text Web site: www.aw.com/bennett-briggs

76. Energy Comparisons. Using data available from the Energy Information Administration Web site, choose a few measures of U.S. or world energy consumption or production. Make comparisons that put these numbers in perspective.

77. Nuclear Fusion. Learn about the current state of research into building commercially viable fusion power plants. What obstacles must still be overcome? Do you think fusion power will be a reality in your lifetime? Explain.


79. Richest People. Find the net worth of the world’s three richest people. Put these monetary values in perspective through any techniques you wish.

In the News

80. Large Numbers. Search today’s newspaper for as many numbers larger than 100,000 as you can find. Briefly explain the context within which each large number is used.

81. Perspective in the News. Find an example in the recent news in which a reporter uses a technique to put a number in perspective. Describe the example. Do you think the technique is effective? Can you think of a better way to put the number in perspective? Explain.

82. Putting Numbers in Perspective. Find at least two examples of very large or small numbers in recent news reports. Use a technique of your choosing to put each number into perspective in a way that you believe most people would find meaningful.

DEALING WITH UNCERTAINTY

The rosy outlook for the United States federal budget was big news in early 2001. Government economists projected a cumulative surplus of $5.6 trillion for the coming 10 years (through 2011). Politicians argued about whether this windfall should be returned to taxpayers, spent on new programs, or used to pay down the federal debt.

A mere two years later, the projected surplus had completely vanished. Instead of surpluses, most economists were predicting that the government would run huge deficits for years to come. How did the $5.6 trillion surplus—some $20,000 for every man, woman, and child in the United States—disappear?

It didn’t really disappear, of course. Rather, it never existed in the first place—it was only an estimate. Like all estimates, it was only as good as the assumptions that went into it, and these assumptions included highly uncertain predictions about the future of the economy, future tax rates, and future spending. The government economists who made the projection were well aware of the uncertainties (and diligently reported these uncertainties in press releases), but the news media and politicians tended to report the surplus projection as an indisputable fact.

This story of vanishing trillions holds an important lesson. Many of the numbers we encounter in daily life are far less certain than we are told, and we can therefore be severely misled unless we learn to examine and interpret uncertainties for ourselves. In this unit, we will discuss ways of dealing with uncertainty properly.
2040 years for the age of the ruins. However, 2000 years is the least precise of the two numbers: It is precise only to the nearest 1000 years, while 40 years is precise to the nearest 10 years. Thus, the answer also should be precise only to the nearest 1000 years:

\[
\begin{align*}
2000 \text{ yr} & + 40 \text{ yr} = 2040 \text{ yr} \\
\text{precise to nearest 1000} & \quad \text{precise to nearest 10} & \quad \text{must round to nearest 1000} & \quad \text{correct final answer}
\end{align*}
\]

To the precision with which we know the age of the ruins, they are still 2000 years old, despite the 40-year age of the book.

b. We find the average tax by dividing the $41.5 million, which has 3 significant digits, by the population of 82,000, which has 2 significant digits. Thus, the answer should have 2 significant digits:

\[
\begin{align*}
$41,500,000 & \div 82,000 \text{ persons} = \frac{\$506.10 \text{ per person}}{2 \text{ significant digits}} \\
\text{3 significant digits} & \quad \text{2 significant digits} & \quad \text{must round to} & \quad \text{2 significant digits} & \quad \text{correct final answer}
\end{align*}
\]

The average resident must pay about $510 in taxes.

Now try Exercises 59–66.

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**EXERCISES 3C**

**Review Questions**

1. What are significant digits? How can you tell whether zeros are significant?

2. Distinguish between random errors and systematic errors. Give an example of how each type might affect measurements of weight. How can we minimize the effects of random errors? How can we account for the effects of a systematic error?

3. Distinguish between the absolute error and the relative error in a measurement. Give an example in which the absolute error is large but the relative error is small and another example in which the absolute error is small but the relative error is large.

4. Distinguish between accuracy and precision. Give an example of a measurement that is precise but inaccurate and another example of a measurement that is accurate but imprecise.

5. Why can it be misleading to give measurements with more precision than is justified by the measurement process?

6. State the rounding rules for adding and subtracting measured numbers and for multiplying and dividing measured numbers. Give examples of their use.

**Does It Make Sense?**

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. Next year’s federal deficit will be $275.734 billion.

8. In many developing nations, official estimates of the population may be off by 10% or more.

9. I weigh 110.3627 pounds.

10. The civil rights rally on the National Mall was attended by 27,503 people.

11. The number of people who registered and paid for this year’s Bolder Boulder (a popular 10-kilometer race) was 48,103.

12. More precision is useless if the measurement is inaccurate.

13. A $1 million error may sound like a lot, but when compared to our company’s revenue it represents a relative error of only 0.1%.

14. Once we corrected the measurements for all known systematic errors, we were sure that our results had perfect accuracy.
Basic Skills & Concepts


15. Round the following numbers to the nearest whole number.
   a. 2.4  
   b. 14.500  
   c. 779.49  
   d. 4.1999  
   e. 13.5001  
   f. 234.5  
   g. 1999.5  
   h. 88.71  
   i. -13.998

16. Round the following numbers to the nearest thousandth, tenth, ten, and hundred.
   a. 2365.98521  
   b. 322354.09005  
   c. 6000  
   d. 3  
   e. 78.555  
   f. 0.4523768  
   g. -12.1  
   h. -850.7654  
   i. -12.001

- **Counting Significant Digits.** In Exercises 17–28, state the number of significant digits and the implied meaning of the given number.
   17. 96.2 km/hr  
   18. 401 people  
   19. 100.020 seconds  
   20. 200.0 liters  
   21. 0.00098 mm  
   22. 1.00098 mm  
   23. 0.0002020 meter  
   24. 0.000202 meter  
   25. 300,000  
   26. 550,000,000  
   27. 3.0000 × 10^3  
   28. 5.500 × 10^6

- **Rounding with Significant Digits.** In Exercises 29–34, carry out the indicated operation and give your answer with the specified number of significant digits.
   29. 1452 × 9076.7; 2 significant digits  
   30. 1452 × 9076.7; 4 significant digits  
   31. 46.371 ÷ 0.00728; 3 significant digits  
   32. 46.371 ÷ 0.00728; 1 significant digit  
   33. (3.4 × 10^5) ÷ (4.1 × 10^-6); 2 significant digits  
   34. (4 × 10^-7) ÷ (9.2489 × 10^-2); 2 significant digits

- **Sources of Errors.** For each measurement described in Exercises 35–42, briefly describe any likely sources of random errors and any likely sources of systematic errors.
   35. A count of sport utility vehicles passing through a busy intersection during a 20-minute period  
   36. The population of a county in Mississippi, according to a census count  
   37. The average income of 25 people, found by checking their tax returns

38. The average income of 25 people, found by asking them their incomes during interviews conducted at a supermarket.

39. Weights of sandwich meats, measured in a delicatessen using a well-calibrated scale.

40. Weights of adults, measured by weighing them with their clothes on.

41. The numbers of popped kernels in "large" boxes of popcorn at a movie theater.

42. Times in a swimming meet.

43. **Tax Audit.** A tax auditor reviewing a tax return looks for several kinds of problems, including these two: (1) mistakes made in entering or calculating numbers on the tax return and (2) places where the taxpayer reported income dishonestly. Discuss whether each problem involves random or systematic errors.

44. **AIDS Epidemic.** Researchers studying the progression of the AIDS epidemic need to know how many people are suffering from AIDS, which they can try to determine by studying medical records. Two of the many problems they face in this research are that (1) some people who are suffering from AIDS are misdiagnosed as having other diseases, and vice versa, and (2) some people with AIDS never seek medical help and therefore do not have medical records. Discuss whether each problem involves random or systematic errors.

45. **Safe Air Travel.** Before taking off, a pilot is supposed to set the aircraft altimeter to the elevation of the airport. A pilot leaves Denver (altitude 5280 ft) with her altimeter set to 2500 feet. Explain how this affects the altimeter readings throughout the flight. What kind of error is this?
46. **Lumber Yard.** A lumber yard is cutting boards that are supposed to be 4 feet long. Each board is cut by an employee who first measures and marks the 4-foot length. Later, careful measurements show that the average length of the boards was indeed 4 feet, but that most of the boards were actually slightly longer or shorter than 4 feet. What type of measurement error is involved in this situation? Explain.

**Absolute and Relative Errors.** In Exercises 47–54, find the absolute and relative errors.

47. The price of a gift you purchase by mail-order is supposed to be $18.50, but instead you are billed for $19.00.

48. The label on a bag of dog food says “20 pounds,” but the true weight is only 18 pounds.

49. The bank is supposed to charge you interest of $65 on your loan, but instead charges you only $48.

50. You cut a piece of linen to a length of 48.5 inches, but the instructions call for a piece that is 48 inches long.

51. The recipe calls for 5 1/2 cups of flour, but you measure 5⅓ cups of flour.

52. The meeting is supposed to last one hour, but it lasts one hour and five minutes.

53. The official distance to Longmont is 12 miles, but your odometer reads 11.7 miles.

54. The diameter of a ball bearing is supposed to be 0.1 centimeter, but instead is 0.2 centimeter.

**Accuracy and Precision.** Exercises 55–58 each describe a pair of measurements. In each case, state which measurement is more precise and which measurement is more accurate. Explain.

55. Your true height is 62.50 inches. A tape measure that can be read to the nearest 1/8 inch gives your height as 62⅜ inches. A new laser device at the doctor’s office that gives readings to the nearest 0.05 inch gives your height as 62.90 inches.

56. Your true height is 62.50 inches. A tape measure that can be read to the nearest 1/8 inch gives your height as 62⅜ inches. A new laser device at the doctor’s office that gives readings to the nearest 0.05 inch gives your height as 62.50 inches.

57. Your weight is 52.55 kilograms. A scale at a health clinic that gives weight measurements to the nearest half kilogram gives your weight as 53 kilograms. A digital scale at the gym that gives readings to the nearest 0.01 kilogram gives your weight as 52.88 kilograms.

58. Your weight is 52.55 kilograms. A scale at a health clinic that gives weight measurements to the nearest half kilogram gives your weight as 52⅓ kilograms. A digital scale at the gym that gives readings to the nearest 0.01 kilogram gives your weight as 51.48 kilograms.

**Combining Measured Numbers.** Use the appropriate rounding rules to answer Exercises 59–66 with the correct precision or correct number of significant digits.

59. Subtract the volumes 140 liters and 1.09 liters.

60. Add the times 2 hours, 37 minutes and 1 hour, 22 minutes, 15 seconds.

61. Multiply the weights 9.7 kilograms and 165 kilograms.

62. Divide the distance 110 kilometers by the time 55 minutes.

63. As you drive down the freeway, a sign tells you that it is 4 miles to the downtown. Your destination lies 2.2 miles beyond city hall (which is downtown). How much farther do you have to drive?

64. At the hardware store, you buy a 50-kilogram bag of nails. You also buy 1.25 kilograms of nails. What is the total weight of your purchases?

65. The government in a city of 480,000 people plans to spend $112.4 million on a transportation project. Assuming all this money must come from taxes, what average amount must the city collect from each resident?

66. Representatives in Congress serve an average of 630,000 constituents. If a political consultant recommends campaign spending of $4.25 per constituent for an upcoming election, how much should a representative spend in?

**Further Applications**

**Believable Facts?** Exercises 67–74 give statements of facts taken from official sources. For each statement, briefly discuss possible sources of error in the measurement. Then, in light of the precision with which the measurement is given, discuss whether you think the fact is believable.

67. The population of the United States in 1860 was 31,443,321.

68. The number of deaths worldwide due to coronary heart disease in 1997 was 7.2 million.
The population of Tokyo in 2015 will be 28.7 million.

The Petrona Towers in Kuala Lumpur are 1483 feet tall, making them the world's tallest buildings as of 2003.

The average maximum temperature in January in Lagos, Nigeria, is 88 degrees.

The number of deaths due to AIDS in the United States as of last year was 390,242.

The number of cell phone users in the United States last year was 55,312,293.

Worldwide in 1998, there were 1694 officially declared endangered or threatened species.

77. Global Warming. Find current estimates regarding how much the Earth has warmed over the past century and how much human activity contributes to that warming. Based on what you learn, write a one- to two-page essay about the uncertainty involved in understanding how human activity is contributing to global warming.

In the News

78. Random and Systematic Errors. Find a recent news report that gives a measured quantity (for example, a report of population, average income, or the number of homeless people). Write a short description of how the quantity was measured, and briefly describe any likely sources of either random or systematic errors. Overall, do you think that the reported measurement was accurate? Why or why not?

79. Absolute and Relative Errors. Find a recent news report that describes some mistake in a measured, estimated, or projected number (for example, a budget projection that turned out to be incorrect). In words, describe the size of the error in terms of both absolute error and relative error.

80. Accuracy and Precision. Find a recent news article that causes you to question accuracy or precision. For example, the article might report a figure with more precision than you think is justified, or it may cite a figure that you know is inaccurate. Write a one-page summary of the report and explain why you question its accuracy or precision (or both).

81. Uncertainty in the News. Look through the newspapers from the past week for articles that mention numbers in any context. Find at least two numbers from each of the following sections: national/international news, local news, sports, and business. In each case, describe the number and its context, and discuss any uncertainty that you think is associated with the number.

82. Real-World Rounding. Find at least three different examples of numbers that were rounded in news stories. Explain how and why each number was rounded. (Hint: You might consider baseball batting averages, times in athletic events, or the stock market.)