**Solution** We can think of the three antigens as three sets $A$, $B$, and $Rh$ (positive) and draw a Venn diagram with three overlapping circles. Figure 1.25 shows the eight regions, each labeled with its type and percentage of the population. For example, the central region corresponds to the presence of all three antigens (AB positive), so it is labeled with 3%. You should check that all eight regions are labeled according to the data from Table 1.2.

**Historical Note**

Human blood groups were discovered in 1901 by Austrian biochemist Karl Landsteiner. In 1909, he classified the groups he had discovered as A, B, AB, and O. He also showed that blood transfusions could be done successfully if the donor and recipient were of the same blood type. For this work, he was awarded the 1930 Nobel Prize in Medicine.

**Exercises 1C**

**Review Questions**

1. What is a set? Describe the use of braces notation for listing the members of a set.
2. What is a Venn diagram? How do we show that one set is a subset of another in a Venn diagram? How do we show disjoint sets? How do we show overlapping sets?
3. List the four standard categorical propositions. Give an example of each type, and draw a Venn diagram for each of your examples.
4. Briefly discuss how you can put a categorical proposition into one of the standard forms if it is not in such a form already.
5. Explain how to draw a Venn diagram for three overlapping sets. Discuss the types of information that can be shown in such diagrams.
6. Explain how to read a table such as Table 1.1 and how to show the information in a Venn diagram.

**Does It Make Sense?**

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. The payments we make to the electric company are a subset of the payments we make to the phone company.
8. I don’t know what you mean by the sets “jabbers” and “wocks,” but I can still draw a Venn diagram for them if you just tell me if and how they share members.
9. I counted an irrational number of students in my class.
10. The number of students in my class is a real number.
11. My professor asked me to draw a Venn diagram for a categorical proposition, but I couldn’t do it because the proposition was clearly false.

12. I used a Venn diagram to prove that your opinion is false.

**Basic Skills & Concepts**

**Classifying Numbers.** In Exercises 13–28, consider the sets natural numbers, whole numbers, integers, rational numbers, and real numbers. Identify from the list the simplest set that describes each number given. Explain.

13. 2.3
14. 23.8
15. \(-\frac{3}{4}\)
16. \(\sqrt{3}\)
17. 3
18. 15
19. \(-5\)
20. \(-25\)
21. 100.1
22. \(\frac{3}{4}\)
23. \(-6.1\)
24. \(-14.1\)
25. \(\sqrt{5}\)
26. \(-\pi/2\)
27. 2005
28. \(-\pi\)

**Set Notation.** In Exercises 29–36, use braces to write the members of each of the following sets, or state that the set has no members. (Dots may also be helpful.)

29. The months of the year
30. The natural numbers from 1 to 100 (inclusive)
31. The first three U.S. presidents
32. The states whose names start with N
33. The states whose names start with C
34. The letters in the second half of the alphabet
35. The natural numbers from 100 to 200 (inclusive)
36. The last three U.S. presidents

**Venn Diagrams for Two Sets.** For Exercises 37–44, draw a Venn diagram for the given sets. In words, explain why you drew one set as a subset of the other, disjoint sets, or overlapping sets.

37. doctors and women
38. reptiles and mammals
39. words and verbs
40. pianists and musicians
41. painters and artists
42. genetics courses and science courses
43. negative integers and natural numbers
44. whole numbers and positive integers

**Categorical Propositions.** Each of Exercises 45–52 gives a categorical proposition. If it is not already in standard form, rephrase in standard form. State the subject and predicate sets, and draw a Venn diagram for the proposition. Label all regions of the diagram clearly.

45. All bachelors are men.
46. Some pilots are women.
47. All U.S. presidents have been men.
48. Some mammals swim.
49. Fish don’t fly.
50. Dogs can’t swim.
51. Every nurse knows CPR.
52. Lawyers are highly educated.

**Venn Diagrams for Three Sets.** For Exercises 53–58, draw a Venn diagram with three overlapping circles for the three given sets. On your diagram, label the contents of every region. If a region has no members, state that fact clearly.

53. women, dentists, and airline pilots
54. basketball players, professional athletes, and men
55. songs, novels, and published works
56. oceans, bodies of salt water, and bodies of fresh water
57. clothes, hats, and shirts
58. states of the United States, places in North America, and places with coastlines on the Pacific Ocean

**A Venn Diagram with Numbers.** In Exercises 59–60, consider the following Venn diagram that represents people in a room. The number in each region is the number of people matching the characteristics of that region.
59. a. How many people are in the room?  
b. How many women are in the room?  
c. How many men are in the room?  
d. How many Republicans are in the room?  

60. a. How many men are Republicans?  
b. How many men are not Republicans?  
c. How many women are Republicans?  
d. How many women are not Republicans?  

**Venn Diagrams for Sets.** In Exercises 61–64, draw a Venn diagram to represent the given information.

61. Of the children on a school bus, 12 play soccer only, 6 play softball only, 2 play soccer and softball, and 4 play neither softball nor soccer.

62. A movie critic rated 15 feature movies and 10 documentaries. He gave 7 feature movies a “thumbs up” and the other feature movies a “thumbs down.” He gave 9 of the documentaries a “thumbs up.”

63. A doctor testing a new antibiotic puts 15 patients on a daily dose of the drug and 15 patients on a daily dose of a placebo (a substance that looks like the actual drug, but has no active ingredients). Twelve of the patients on the actual drug show improvement after a week, while 6 of the patients on the placebo show improvement.

64. A survey of 100 Toyota and Nissan owners revealed that all owners drove one vehicle and it was either an SUV or a pickup. Half of the 60 Toyota owners drove SUVs, while one-fourth of the Nissan owners drove SUVs.

**Further Applications**

**Venn Diagram Answers.** In Exercises 65–66, draw a two-circle Venn diagram that contains all the listed information. Label each region clearly. Then use the diagram to answer the questions.

65. Of the 43 students in the Cosmos Club, 16 are taking at least mathematics, 23 are taking at least biology, and 9 are taking mathematics only. How many students are taking biology only? How many are taking neither biology nor mathematics?

66. Of the 17 students in the Cycling Club, 12 own at least a road bike, 8 own at least a mountain bike, and 0 own no bike. How many students own both types of bike?

67. **Majors and Gender.** A scan of all biology and business majors at a college shows the following breakdown by gender. Draw a Venn diagram for the data.

<table>
<thead>
<tr>
<th></th>
<th>Biology</th>
<th>Business</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women</td>
<td>32</td>
<td>110</td>
</tr>
<tr>
<td>Men</td>
<td>21</td>
<td>87</td>
</tr>
</tbody>
</table>

68. **Tomatoes and Cancer.** A 1999 study by the Harvard Medical School (Journal of the National Cancer Institute, February 17, 1999) reviewed 72 previous studies of the effect of tomatoes on cancer. The data showed convincingly that “high consumers of tomatoes and tomato products are at substantially decreased risk of numerous cancers, although probably not all cancers.” Consider the following table that shows the incidence of oral cancer for a group of people who ate an average of one tomato a day and another group of people who ate fewer than three tomatoes per week. Draw a Venn diagram for the data.

<table>
<thead>
<tr>
<th></th>
<th>No oral cancer</th>
<th>Oral cancer</th>
</tr>
</thead>
<tbody>
<tr>
<td>One tomato per day</td>
<td>191</td>
<td>9</td>
</tr>
<tr>
<td>Fewer than three tomatoes per week</td>
<td>164</td>
<td>16</td>
</tr>
</tbody>
</table>

69. **Coffee and Gall Stones.** A study on the effect of coffee on gall stones (Journal of the American Medical Association, June 9, 1999) resulted in small part in the data shown below. The category Coffee means more than four cups of caffeinated coffee per day. The category No coffee means no caffeinated coffee. Draw a Venn diagram for the data.

<table>
<thead>
<tr>
<th></th>
<th>Gall stone disease</th>
<th>No disease</th>
</tr>
</thead>
<tbody>
<tr>
<td>No coffee</td>
<td>385</td>
<td>14,068</td>
</tr>
<tr>
<td>Coffee</td>
<td>91</td>
<td>4,806</td>
</tr>
</tbody>
</table>

70. **Interpreting a Survey.** A survey asked newspaper readers which of the following three newspapers they read daily: New York Times (NYT), Washington Post (WP), and Wall Street Journal (WSJ). The results are shown below. Answer the following questions, interpreting the connecting words carefully.

<table>
<thead>
<tr>
<th>Paper(s)</th>
<th>Readers</th>
<th>Paper(s)</th>
<th>Readers</th>
</tr>
</thead>
<tbody>
<tr>
<td>NYT only</td>
<td>24</td>
<td>NYT and WSJ only</td>
<td>14</td>
</tr>
<tr>
<td>WSJ only</td>
<td>27</td>
<td>NYT and WP only</td>
<td>16</td>
</tr>
<tr>
<td>WP only</td>
<td>26</td>
<td>WP and WSJ only</td>
<td>13</td>
</tr>
<tr>
<td>None</td>
<td>15</td>
<td>All three</td>
<td>8</td>
</tr>
</tbody>
</table>
71) Interpreting Sales. A review of sales at a large automobile dealership for the past month shows that the following numbers of buyers chose air conditioning (AC), four-wheel-drive (4WD), and CD players (CD) in their cars.

<table>
<thead>
<tr>
<th>Option(s)</th>
<th>Buyers</th>
<th>Option(s)</th>
<th>Buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC only</td>
<td>45</td>
<td>AC and 4WD only</td>
<td>21</td>
</tr>
<tr>
<td>4WD only</td>
<td>8</td>
<td>AC and CD only</td>
<td>16</td>
</tr>
<tr>
<td>CD only</td>
<td>12</td>
<td>CD and 4WD only</td>
<td>8</td>
</tr>
<tr>
<td>None</td>
<td>3</td>
<td>All three</td>
<td>10</td>
</tr>
</tbody>
</table>

a. Draw a Venn diagram for the table.
b. How many people chose AC and 4WD?
c. How many people chose AC but not 4WD?
d. How many people chose AC or 4WD or CD?
e. How many people chose exactly two options?

72) Analyzing Diets. The following table shows ten common foods and whether they are significant sources of protein, fat, and carbohydrate. Organize these data in a Venn diagram. (Hint: You'll need circles for protein, fat, and carbohydrate; then put the name of each food in the appropriate region of the diagram.)

<table>
<thead>
<tr>
<th>Food</th>
<th>Protein</th>
<th>Fat</th>
<th>Carbohydrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Rice</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Nuts</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Potatoes</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Apples</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Cheese</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Eggs</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Lettuce</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Organizing Propositions. Each of Exercises 77–79 lists a set of propositions. Draw a Venn diagram that represents all the information in the propositions and use it (and no other assumptions) to answer the questions that follow. Explain your reasoning.

77. All hairy animals are mammals. No mammals are fish. Some mammals can swim. No fish can walk on land.

Questions: Could there be hairy fish? Could there be hairy animals that swim? Could there be walking mammals? Could there be hairy animals that walk on land?

78. All meat has protein. All dairy products have protein. Some beans have protein. All beans, but no meat or dairy products, are plants.

Questions: Could there be beans that are dairy products? Could there be meat that is a dairy product? Could there be dairy products that are plants? Could there be plants with protein?

79. No Republicans are Democrats. No Republicans are Green Party members. All Republicans are conservative. Some liberals are Democrats. No liberals are conservatives.

Questions: Could there be conservative Democrats? Could there be liberal Green Party members? Could there be liberal Republicans?

80. Organizing Politicos. You are at a conference attended by men and women of various political parties. The conference organizer tells you that none of the women are Republicans and some (but not all) of the Democrats are women.

a. Draw a Venn diagram to organize the given information.
b. Based on the given information, is it possible to meet a woman who is neither a Republican nor a Democrat?
c. Based on the given information, is it possible that there are any male Republicans?

81. Organizing Literature. In reviewing for an exam in your literature survey course, you notice the following facts about the writers that you studied:

* Some of the novelists are also poets.
* None of the novelists are playwrights.
• All of the novelists were born in the 20th century.
• All of the writers born in the 19th century are playwrights.
  a. Organize these facts in a Venn diagram.
  b. Could you have studied a novelist born in the 19th century?
  c. Could you have studied a poet born in the 19th century?
  d. Could you have studied a writer born in the 20th century who was both a playwright and a poet?

Is It Possible? Shoppers are given a choice of one of three brands of vanilla ice cream and then asked if they approve or disapprove of the flavor. Is it possible to draw a Venn diagram with two circles that shows how many shoppers approved or disapproved of each brand? Explain.

N Sets. A computer store is offering a basic computer with four options A, B, C, and D, each of which buyers can select.
  a. How many different sets of options can buyers choose? (For example, one choice is no options, another choice is only options A and C, and another choice is all options.)
  b. Is it possible to make a Venn diagram using circles to show the number of buyers selecting each of the choices?
  c. Suppose the store offered five options A, B, C, D, and E. How many different sets of options would be available?
  d. Can you generalize parts a–c? If the store offers n options, each of which can be either selected or not selected, how many different sets of options are available?

Web Projects

Find useful links for Web Projects on the text Web site: www.aw.com/bennett-briggs

84. State Politics. Determine from the Web how many states have a Republican majority in the State House and how many states have a Republican majority in the State Senate. Draw a Venn diagram to illustrate the situation.

85. U.S. Presidents. Use the Web to learn the following about each past American president:
  • Bachelor or married (classify as married if married for part of the term)
  • Inaugurated before or after age 50
  • Served one term (or less) or more than one term
Make a three-circle Venn diagram to represent your results.

86. Uses of Venn Diagrams. Visit the Web site for VennDiagram.com and read about the top 10 uses of Venn diagrams. Choose at least one of these uses and apply it to a specific problem in your own life.

In the News

87. Categorical Propositions. Find at least three examples of categorical propositions in news articles or advertisements. State the sets involved in each proposition and draw a Venn diagram for each proposition.

88. Venn Diagrams in Your Life. Describe a situation in your own life that could be described or organized using a Venn diagram.

89. Quantitative Diagram. Find a news article or research report with a table similar to Table 1.1 or 1.2. Draw a Venn diagram to represent the data in the table.


**EXERCISES 1D**

**Review Questions**

1. Summarize the differences between deductive and inductive arguments. Give an example of each type.

2. Briefly explain the idea of strength and how it applies to inductive arguments. Can an inductive argument prove its conclusion true? Can an inductive argument be valid? Can it be sound?

3. Briefly explain the ideas of validity and soundness and how they apply to deductive arguments. Can a valid deductive argument be unsound? Can a sound deductive argument be invalid? Explain.

4. Describe the procedure used to test the validity of a deductive argument with a Venn diagram.

5. Create your own example of each of the four basic conditional arguments. Then explain why your argument is valid or invalid.

6. What is a chain of conditionals? Give an example of a valid argument made from such a chain.

7. Can inductive logic be used to prove a mathematical theorem? Explain.

8. How can inductive testing of a mathematical rule be useful? Give an example.

9. The many examples of people whose cancer went away following chemotherapy make a strong case for the idea that chemotherapy can cure cancer.

10. Through the logic of deduction, I will show you that if you accept the truth of my premises, you must also accept the truth of my conclusion.

11. You can see that my argument is valid, and you must therefore accept the truth of my conclusion.

12. We should never accept an argument unless its conclusion has been proven true beyond all doubt.

13. Even before Fermat's Last Theorem was proved deductively, mathematicians were sure it was true.

**Does It Make Sense?**

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

9. Based on the testimonials of dozens of people who have lost weight following my diet, I will prove to you that my diet works for everyone.

10. I see the same tall man leaving that office building every day at 5:00. He must work in the building.

11. If I carry a heavy load in the trunk of my car, the rear wheels squeak. If the rear wheels squeak, then I get a headache. Therefore, if I carry a heavy load in the trunk of my car, I get a headache.

12. Scott Turow's first five novels were great. I'm sure his next novel will be great too.

13. I've never had a bad meal at Dot's Diner, so I recommend it to everyone.

14. Any restaurant that gives free refills of coffee is OK by me, and since Fred's Food gives free refills, I'll go there any day.

15. When the wind blows, the garden dries out, and when the garden dries out, I need to water it. So whenever the wind blows, I can expect to water the garden.
21. The last four times I went skiing, the traffic was light on Tuesdays and heavy on Saturdays. Weekdays must have lighter traffic than weekends.

22. I like Chinese cuisine and I like Thai cuisine, so I'd also like any other Asian cuisine.

Analyzing Inductive Arguments. For Exercises 23–28, determine the truth of the premises, discuss the strength of the argument, and assess the truth of the conclusion.

23. Premise: Cows have four limbs, and they are mammals.
   Premise: Monkeys have four limbs, and they are mammals.
   Premise: Lions have four limbs, and they are mammals.
   Conclusion: All animals with four limbs are mammals.

24. Premise: \((-6) \times (-4) = 24\)
   Premise: \((-2) \times (-1) = 2\)
   Premise: \((-27) \times (-3) = 81\)
   Conclusion: Whenever we multiply two negative numbers, the result is a positive number.

25. Premise: Grant Hill wears expensive shoes, and he is a great basketball player.
   Premise: Tim Duncan wears expensive shoes, and he is a great basketball player.
   Premise: Shaquille O'Neal wears expensive shoes, and he is a great basketball player.
   Conclusion: All people who wear expensive shoes are great basketball players.

26. Premise: Bach, Buxtehude, Beethoven, Brahms, Berlioz, and Britten are great composers.
   Conclusion: All great composers have names that begin with B.

27. Premise: Students from Ms. Callahan's Algebra I class do better in Algebra II than students from Mr. McIntyre's Algebra I class.
   Premise: Students from Ms. Callahan's Algebra I class do better in Algebra II than students from Mr. Murphy's Algebra I class.
   Conclusion: Ms. Callahan is the best of the three Algebra I teachers.

28. Premise: Sparrows are birds and they fly.
   Premise: Eagles are birds and they fly.
   Premise: Hawks are birds and they fly.
   Premise: Larks are birds and they fly.
   Conclusion: All birds fly.

Deductive Arguments. For Exercises 29–36, start by rephrasing the argument, if necessary, so that the first premise has the form all S are P or no S is P. Then draw a Venn diagram to determine whether the argument is valid. If possible, discuss the truth of the premises and whether the argument is sound.

29. Premise: All islands are tropical.
   Premise: Iceland is an island.
   Conclusion: Iceland is tropical.

30. Premise: All dairy products contain protein.
   Premise: Soybeans contain protein.
   Conclusion: Soybeans are dairy products.

31. Premise: All salty foods cause high blood pressure.
   Premise: Apples do not cause high blood pressure.
   Conclusion: Apples are not salty foods.

32. Premise: All U.S. presidents have been men.
   Premise: George Washington was a man.
   Conclusion: George Washington was a U.S. president.

33. Premise: All states in the Eastern Standard time zone are east of the Mississippi River.
   Premise: Maine is in the Eastern Standard time zone.
   Conclusion: Maine is east of the Mississippi River.

34. Premise: Doctors know anatomy.
   Premise: Cardiologists know anatomy.
   Conclusion: Cardiologists are doctors.

35. Premise: All opera singers can whistle a Mozart tune.
   Premise: Pavarotti is an opera singer.
   Conclusion: Pavarotti can whistle a Mozart tune.

36. Premise: No movie stars do their own laundry.
   Premise: Jack Applebee does his own laundry.
   Conclusion: Jack Applebee is not a movie star.
Conditional Deductive Arguments. For Exercises 37–44, start by rephrasing the argument, if necessary, so that the first premise is in the conditional form \( \text{if } p, \text{ then } q \). Identify the type of argument and determine its validity with a Venn diagram. If possible, discuss the truth of the premises and whether the argument is sound.

37. Premise: If an animal is a horse, then it is a mammal.
   Premise: Clydesdales are horses.
   Conclusion: Clydesdales are mammals.

38. Premise: In the United States, we have the right to say anything at any time.
   Premise: Yelling "Fire!" in a theater is saying something.
   Conclusion: In the United States, we have the right to yell "fire!" in a theater.

39. Premise: If I eat breakfast, then I don't eat lunch.
   Premise: I ate breakfast.
   Conclusion: I didn't eat lunch.

40. Premise: If I eat breakfast, then I don't eat lunch.
   Premise: I didn't eat lunch.
   Conclusion: I ate breakfast.

41. Premise: If I don't eat breakfast, then I eat lunch.
   Premise: I ate breakfast.
   Conclusion: I didn't eat lunch.

42. Premise: It's necessary for nurses to know CPR.
   Premise: Tom is a nurse.
   Conclusion: Tom knows CPR.

43. Premise: When interest rates decline, the bond market improves.
   Premise: Last week the bond market improved.
   Conclusion: Interest rates must have declined.

44. Premise: If we can put a man on the moon, we can build a computer operating system that works.
   Premise: We can build a computer operating system that works.
   Conclusion: We can put a man on the moon.

Chains of Conditionals. For Exercises 45–48, start by rephrasing the given propositions, if necessary, in \( \text{if } p, \text{ then } q \) form. Then write the argument as a chain of conditionals and evaluate its validity.

45. Premise: If you shop, I make dinner.
   Premise: If I make dinner, you take out the trash.
   Conclusion: If you shop, you take out the trash.

46. Premise: If fish have fins, then fish can fly.
   Premise: If fish can fly, then dogs can bark.
   Conclusion: If fish have fins, then dogs can bark.

47. Premise: If taxes are cut, then taxpayers will have more disposable income.
   Premise: With more disposable income, taxpayer spending will fuel the economy.
   Conclusion: A tax cut will fuel the economy.

48. Premise: If taxes are cut, the U.S. government will have less revenue.
   Premise: If there is less revenue, then the deficit will be larger.
   Conclusion: Tax cuts will lead to a larger deficit.

Testing Mathematical Rules. For Exercises 49–52, test the statement with several different numbers and state whether you think the statement is true.

49. Is it true for all numbers \( a \) and \( b \) that \( a + b = b + a \)?
50. Is it true for all numbers \( a \) and \( b \) that \( a + b = b + a \)?
51. Is it true for all numbers \( a \) and \( b \) that \( a^2 + b^2 = (a + b)^2 \)?
52. Is it true for all positive integers \( n \) that
   \[ 1 + 2 + 3 + \cdots + n = \frac{n \times (n + 1)}{2}. \]

Further Applications

Validity and Soundness. For Exercises 53–57, state whether it is possible for a deductive argument to have the given properties. If so, make up an example that has these properties.

53. Valid and sound
54. Not valid and sound
55. Valid and not sound
56. Valid with false premises and a true conclusion
57. Not valid with true premises and a true conclusion

**Your Own Conditional Arguments.** For Exercises 58–61, create a simple three-line argument of the given form. Choose your example so that it illustrates clearly whether or not the argument is valid.

58. Affirming the hypothesis

59. Affirming the conclusion

60. Denying the hypothesis

61. Denying the conclusion

62. The Goldbach Conjecture. Recall that a prime number is any number whose only factors are itself and 1 (for example, 2, 3, 5, 7, 11, . . .). The Goldbach conjecture, posed in 1742, claims that every even number greater than 2 can be expressed as the sum of two primes. For example, 4 = 2 + 2, 6 = 3 + 3, and 8 = 5 + 3. A deductive proof of this conjecture has never been found. Test the conjecture for at least 10 even numbers and present an inductive argument for its truth. Do you think the conjecture is really true? Why or why not?

**Web Projects**

Find useful links for Web Projects on the text Web site: www.aw.com/bennett-briggs

63. Fermat's Last Theorem. One of the most famous mathematical theorems of all time is called Fermat's Last Theorem. For over 350 years, the proof eluded mathematicians; it was finally cracked by Andrew Wiles in the 1990s. Through Web research, learn what the theorem claims and why it became so famous. Briefly summarize how inductive evidence suggested its truth for centuries before Wiles finally found a deductive proof.

64. The Pythagorean Theorem. Learn more about the history of the Pythagorean theorem, and write a short report on one aspect of its history. For example, you might write about its use in cultures that predated Pythagoras or describe another proof of the theorem with its historical context.

**In the News**

65. Deductive Reasoning in Your Life. Give an example of a situation in which you used deductive reasoning in everyday life. Explain the situation, describe the steps in your thinking, and explain why it was deductive reasoning.

66. Inductive Reasoning in Your Life. Give an example of a situation in which you used inductive reasoning in everyday life. Explain the situation, describe the steps in your thinking, and explain why it was inductive reasoning.

67. Editorial Arguments. Find three simple arguments in editorials. State whether each is deductive or inductive and evaluate it accordingly.

68. Arguing Your Side. Choose an issue that you feel strongly about and create an argument in support of your position. Is your argument inductive or deductive? Evaluate your argument.

69. Arguing the Other Side. Choose an issue that you feel strongly about and create an argument that tends to contradict your position. That is, try to create an argument for the other side. Can you make the argument convincing? Does the argument help you understand the other side of the issue?

**CRITICAL THINKING IN EVERYDAY LIFE**

The skills discussed in the preceding units are all useful in their own right. But critical thinking involves much more than isolated skills. It also involves knowing when to use particular skills and when to invent entirely new strategies to analyze an argument or make a decision. Indeed, as we stated at the very beginning of this chapter, critical thinking involves careful reading (or listening), sharp thinking, logical analysis, good visualization, and healthy skepticism.

Because it is so wide-ranging, critical thinking cannot be described by any simple step-by-step procedure. Instead, it is developed through experience and by questioning
EXERCISES 2A

**Review Questions**

1. What are units? Describe how to read units that involve division, multiplication, squares, and cubes.

2. Explain why a unit conversion really involves just multiplying by 1.

3. Describe the three forms in which we can write any conversion factor. If given the conversion in one form, such as 1 lb = 16 oz, how do you find the other two forms?

4. Explain in words or with a picture why there are 9 square feet in 1 square yard and 27 cubic feet in 1 cubic yard. Then describe generally how to find conversion factors involving squares or cubes.

5. Describe how to read and use the currency data in Table 2.1.

6. Briefly describe how units can help you check your answers and solve problems. Give examples.

**Does It Make Sense?**

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. I drove really fast—my speed was 50 miles.

8. Our house has a floor area of 1500 square feet.

9. I have a box with a volume of two square feet.

10. I needed five acres of water to fill the swimming pool.

11. I figured out how long the airplane will take to reach Beijing by dividing the airplane’s speed by the distance to Beijing.

12. I figured out how long the airplane will take to reach Beijing by dividing the distance to Beijing by the airplane’s speed.

**Basic Skills & Concepts**


**13.** Evaluate each of the following.

- a. \( \frac{4}{3} \times \frac{1}{2} \)
- b. \( \frac{4}{3} + \frac{1}{2} \)
- c. \( \frac{4}{3} - \frac{1}{2} \)
- d. \( \frac{4}{3} \times 2 \)
- e. \( \frac{7}{20} + \frac{3}{5} \)
- f. \( \frac{12}{13} - \frac{1}{4} \)
- g. \( \frac{7}{17} \times \frac{2}{7} \)
- h. \( \frac{1}{3} + \frac{1}{5} \)

**14.** Evaluate each of the following.

- a. \( \frac{14}{5} - \frac{3}{10} \)
- b. \( \frac{5}{9} + \frac{4}{9} \)
- c. \( \frac{2}{3} + \frac{1}{4} \)
- d. \( \frac{1}{2} + \frac{1}{3} \)
- e. \( \frac{1}{6} \times \frac{6}{11} \)
- f. \( \frac{1}{8} - \frac{1}{16} \)
- g. \( \frac{3}{5} + \frac{5}{3} \)
- h. \( \frac{1}{2} \times \frac{1}{3} \)

**15.** Write each of the following as a common fraction.

- a. 0.3
- b. 0.124
- c. 0.78
- d. 0.005
- e. 1.84
- f. 3.009
- g. 0.0001
- h. 0.1001

**16.** Write each of the following as a common fraction.

- a. 0.7
- b. 1.4
- c. 0.45
- d. 0.988
- e. 0.401
- f. 1.001
- g. 3.07
- h. 0.0002

**17.** Convert each of the following common fractions into decimal form. If necessary, round to the nearest thousandth.

- a. 0.3
- b. 0.4
- c. 0.1
- d. 0.7
- e. 0.9
- f. 0.7
- g. 0.88
- h. 0.122

**18.** Convert each of the following common fractions into decimal form. If necessary, round to the nearest thousandth.

- a. \( \frac{7}{12} \)
- b. \( \frac{27}{45} \)
- c. \( \frac{24}{18} \)
- d. \( \frac{5}{6} \)
- e. \( \frac{537}{612} \)
- f. \( \frac{243}{81} \)
- g. \( \frac{5}{6} \)
- h. \( \frac{16}{3} \)

**Identifying Units.** In Exercises 19–28, identify the units you would expect for the given quantity. State the units both in words (e.g., dollars per gallon) and mathematically (e.g., $/gal).

**19.** The price of apples, found by dividing their total cost in dollars by their total weight in pounds.

**20.** A speed, found by dividing a distance measured in kilometers by a time measured in seconds.

**21.** The installation cost of floor tile, found by dividing the total cost of the installation in dollars by the area of the room in square feet.

**22.** The flow rate of a river in which 5000 cubic feet of water flow past a particular location every second.

**23.** The atmospheric pressure, measured by finding the weight of atmosphere (in pounds) above each square inch of surface on the ground.

**24.** The price of fabric, found by dividing its cost in dollars by its area in square feet.
25. The gas mileage of a car, found by dividing the distance in miles that it travels by the amount of gas in gallons that it uses.
26. The cost for grass seed when you buy enough to cover 80 square yards at a total price of $160.
27. The density of a rock, found by dividing its weight in grams by its volume in cubic centimeters.
28. A car engine torque, calculated by multiplying a force in pounds by a distance in feet.

**Unit Conversions.** In Exercises 29–38, carry out the indicated unit conversion.

29. Convert a distance of 18 yards into feet.
30. Convert a distance of 18 yards into inches.
31. Convert a distance of 7 miles into yards
   \(1760 \text{ yd} = 1 \text{ mi}\).
32. Convert a weight of 142 ounces into pounds
   \(16 \text{ oz} = 1 \text{ lb}\).
33. Convert a lot size of \(\frac{1}{4}\) acre to square feet
   \(1 \text{ acre} = 43,560 \text{ ft}^2\).
34. Convert a park size of 3.5 square miles to acres
   \(1 \text{ acre} = \frac{1}{640} \text{ mi}^2\).
35. Using the fact that there are 1760 yards in a mile and 3 feet in a yard, convert a distance of 3 miles into feet.
36. There are 8 ounces in a cup, 4 cups in a quart, and 4 quarts in a gallon. Using a chain with these conversions, convert 6 gallons into ounces.
37. Use a chain of conversions with familiar measures of time to convert 4 weeks into minutes.
38. A car is driving at 100 kilometers per hour. What is its speed in kilometers per second?

**Area and Volume Calculations.**

a. A swimming pool 3 meters deep, 10 meters long, and 5 meters wide is filled with water. What is the area of the water’s surface? What volume of water does the pool contain?

b. A box measures 22 inches by 15 inches by 12 inches. What is the area of its largest sides? What is the volume of the box?

c. A skyscraper is 1000 feet high with a 25,000-square-foot base. What is its total volume?

**Currency Conversions.** Answer the questions in Exercises 49–56 using the exchange rates in Table 2.1.

49. Which is worth more, 1 British pound or 1 dollar? Explain.
50. Which is worth more, 1 Mexican peso or 1 Japanese yen? Explain.
51. You return from a trip with 2500 Mexican pesos. How much are your pesos worth in dollars?
52. You return from a trip with 75 British pounds. How much are your pounds worth in dollars?
53. How many Japanese yen can you buy for $100?
54. How many Canadian dollars can you buy for $100?
55. Gasoline in Germany sells for about 5.50 euros per gallon. What is this price in dollars per gallon?
56. Apples in Japan sell for about 75 yen each. If you buy 4 apples, how much have you spent in dollars?
Working with Units. Use units to help you answer the questions in Exercises 57–70.

57. A car travels 14 miles in 15 minutes. How fast is it going in miles per hour?
58. An airplane travels 95 miles in 10 minutes. How fast is it going in miles per hour?
59. You purchased 40 acres of farm land for $200,000. How much did you pay per acre?
60. A 40-acre orchard produces 12,000 apples. What is its yield in apples per acre?

61. You are buying 4.7 pounds of apples priced at $1.29 per pound. How much will you pay?
62. You are buying 2.8 kilograms of cherries priced at $3.50 per kilogram. How much will you pay?
63. You need 10 square yards of cloth priced at $2 per square yard. How much will the cloth cost?
64. You are buying floor tile to cover a room that measures 20 feet by 25 feet. The tile is priced at $7.50 per square foot. How much will the tile cost?
65. You are buying carpet to cover a floor area of 1520 square feet. The carpet costs $18 per square yard. How much will the carpet cost?
66. You are buying artificial turf to cover a game field that is 150 feet long and 100 feet wide. The turf costs $7.50 per square yard. How much will the turf cost?
67. You take a trip in which you drive 1200 miles in 20 hours. What is your average speed for the entire trip?
68. You work 40 hours per week and are paid $13.50 per hour. If you work all 52 weeks in a year, how much will you earn?
69. If you sleep an average of 7.5 hours each night, how many hours do you sleep in a year?

70. A human heart beats about 60 times per minute. If an average human being lives to the age of 75, how many times does the average heart beat in a lifetime?

What Went Wrong? Exercises 71–74 include an exam question and a solution given by a student. State whether each solution is right or wrong. If it is wrong, write a note to the student explaining why the answer is wrong and how to solve the problem correctly.

71. Exam Question: A candy store sells chocolate for $7.70 per pound. The piece you want to buy weighs 0.11 pound. How much will it cost, to the nearest cent? (Neglect sales tax.)
   Student Solution: $0.11 \div 7.70 = 0.014.$ It will cost $0.14.

72. Exam Question: You ride your bike up a steep mountain road at 5 miles per hour. How far do you go in 3 hours?
   Student Solution: $5 \div 3 = 1.7.$ I rode 1.7 miles.

73. Exam Question: You can buy a 50-pound bag of flour for $11 or you can buy a 1-pound bag for $0.39. Compare the per pound cost for the large and small bags.
   Student Solution: The large bag price is $50 \div $11 = $4.64 per pound, which is much more than the 39¢ per pound price of the small bag.

74. Exam Question: The average person needs 1500 Calorie day. A can of Coke contains 140 Calories. How many Co would you need to drink to fill your daily caloric needs? (Note: This diet may not meet other nutritional needs!)
   Student Solution: $1500 \times 140 = 210,000.$ You would need to drink 210,000 Cokes to meet your daily caloric needs.

Gas Mileage. Exercises 75–78 involve practical gas mileage calculations.

75. Suppose you drive a car with an average gas mileage of 28 miles per gallon. If you plan to take a 2500-mile cro
country trip, how many gallons of gasoline should you expect to use?

76. Suppose you drive a car with an average gas mileage of 35 miles per gallon. If you plan to take a 2500-mile cross-country trip, how much should you budget (in dollars) for gasoline if the price of gasoline averages $1.65 per gallon?

77. Gas mileage actually varies slightly with the driving speed of a car (as well as with highway vs. city driving). Suppose your car averages 38 miles per gallon on the highway if your average speed is 55 miles per hour and averages 32 miles per gallon on the highway if your average speed is 70 miles per hour.
   a. What is the driving time for a 2000-mile trip if you drive at an average speed of 55 miles per hour? What is the driving time at 70 miles per hour?
   b. Assume a gasoline price of $1.65 per gallon. What is the gasoline cost for a 2000-mile trip if you drive at an average speed of 55 miles per hour? What is the gasoline cost at 70 miles per hour?

78. Suppose your car averages 32 miles per gallon on the highway if your average speed is 60 miles per hour and averages 25 miles per gallon on the highway if your average speed is 75 miles per hour.
   a. What is the driving time for a 1500-mile trip if you drive at an average speed of 60 miles per hour? What is the driving time at 75 miles per hour?
   b. Assume a gasoline price of $1.80 per gallon. What is the gasoline cost for a 1500-mile trip if you drive at an average speed of 60 miles per hour? What is the gasoline cost at 75 miles per hour?

79. Filling a Pool. A swimming pool is 75 feet long and 54 feet wide. It is only partially filled with water, so the water surface is 6 inches below where it is supposed to be. How much water would it take, in cubic feet, to raise the water level by 6 inches?

Further Applications

80. House Footprint. A local zoning ordinance says that a house's "footprint" (area of its ground floor) cannot occupy more than \( \frac{1}{3} \) of the lot it is built on. Suppose you own a 1-acre lot. What is the maximum allowed footprint for your house, in square feet (1 acre = 43,560 ft\(^2\))?  

81. Full of Hot Air. The average person breathes 6 times per minute (at rest), inhaling and exhaling half a liter of air each time. How much "hot air" (the air is warmed by the body), in liters, does the average person exhale each day?

82. E-Books. Computer memory is measured in units of bytes, where one byte is enough memory to store one character (a letter in the alphabet or a number). How many typical pages of text can be stored on a 500-megabyte hard drive (a megabyte is one million bytes)? Assume 2000 bytes per page.

83. Dog Years. Sometimes the age of dogs is described in a unit called "dog years." A commonly used conversion is that 1 real year equals 7 dog years.
   a. If your dog is 15 real years old, what is her age in dog years?
   b. People often refer to the third year in the life of a human child as the "terrible twos" stage. If dogs have a terrible twos stage in their third dog year, how old are they, in real time, during this stage?
   c. Based on what you know about dogs, do you think the common conversion of 1 real year to 7 dog years seems reasonable? Explain.

84. Glen Canyon Flood. The Department of the Interior released a "spike flood" from the Glen Canyon Dam on the Colorado River. Its purpose was to restore the river and the habitats along its banks, particularly in the Grand Canyon. The reservoir behind the dam contains about 1.2 trillion (1,200,000,000,000) cubic feet of water. The release from the dam lasted a week at a rate of 25,800 cubic feet of water per second. About how much water was released during the 1-week flood? What fraction of the total water in the reservoir was released during the flood?

85. Home Project. Assume you are building a simple shed and doing some landscaping. The shed will be the shape...
of a box that is 10 feet long, 10 feet wide, and 8 feet tall. The area you are landscaping is 75 feet long and 40 feet wide. For each of the following questions, find the price of the needed item at a local store.

a. How much would it cost to plant the entire landscaping area with grass seed? (Hint: Grass seed is usually rated by how many square feet can be covered by each pound of seed.)

b. Suppose you decide to cover the landscaping area with sod (i.e., rolls of pre-grown grass). How much will the sod cost?

c. You decide that before any planting, you need to add 8 inches of high-quality top soil to the entire landscape area. How much will the soil cost?

d. How much will it cost to paint the exterior walls of the shed? (Hint: Paint cans generally tell you how much area can be covered with the paint they contain.)

Web Projects

Find useful links for Web Projects on the text Web site: www.aw.com/bennett-briggs

86. South American Adventure. Suppose you are planning an extended trip through many countries in South America. Use one of the many currency exchange sites on the Web to get all the exchange rates you’ll need. Make a brief table showing each currency you’ll need and the value of each currency in dollars.

87. Polar Ice Melting. Starting with a search on “glaciers” or “glaciology” (the study of glaciers), use the Web to learn more about polar ice melting. Focus on one aspect of the issue, such as whether global warming is causing melting, or the environmental impacts of melting, or the geological history of ice ages. Write a one-page summary of what you learn.

In the News

88. Are the Units Clear? Find a news story that involves numerical data. Are all the numbers in the story given with meaningful units, or is the meaning of some of the units unclear? Briefly summarize how well (or not well) the article uses units.

89. Units on the Highway. Next time you are on the highway, look for three signs that use numbers (such as speed limits or distances to nearest exits). Are the units of the numbers given? If not, how are you expected to know the units? In cases where the units are not given, do you think the units would be obvious to everyone? Why or why not?

90. False Advertising? A Goodyear tire commercial began by stating that the Goodyear Aquatread tire can “churn away” 1 gallon of water per second. The announcer then said: “One gallon per second—that’s 396 gallons per minute.” What’s wrong with this statement? What point do you think the advertisement was trying to make? Can you find other examples of advertisements that misstate units?

STANDARDIZED UNITS: MORE PROBLEM-SOLVING POWER

We’ve seen that working with units can be a powerful problem-solving technique—but only if the units are clear and meaningful. When units (such as feet, miles, or kilograms) are standardized, everyone agrees on their meaning. We will now review the two systems of standardized units in common use—the U.S. system and the international metric system—and how to convert between them. We’ll also explore standardized units for temperature, energy, density, and concentration, which will allow us to study a much broader range of problems.
Time out to think
Many college students have lost their lives by rapidly consuming several “shots” of strong alcoholic drinks. Explain why such rapid consumption of alcohol can lead to death, even when the total amount of alcohol consumed may not sound like a lot.

EXERCISES 2B

Review Questions

1. Briefly describe the origin and use of common units in the U.S. customary system.
2. Briefly describe the origin and use of metric units. What are the basic metric units of length, mass, time, and volume? How are the metric prefixes used?
3. Using examples, show how to convert among the Fahrenheit, Celsius, and Kelvin temperature scales.
4. What is energy? List at least three common units of energy. Under what circumstances do the different units tend to be used?
5. What is the difference between energy and power? What are the standard units for power?
6. What do we mean by density? What do we mean by concentration? Describe common units of density and concentration, including blood alcohol content, with examples.

Does It Make Sense?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning. Hint: Be sure to consider whether the units are appropriate to the statement, as well as whether the stated amount makes any sense. For example, a statement that someone is 15 feet tall uses the units (feet) appropriately, but does not make sense because no one is that tall.

7. I drank 2 liters of water today.
8. I know a professional bicyclist who weighs 300 kilograms.
9. Today I drove along the interstate at 100 kilometers per hour.
10. I know someone who can run 35 liters per second.
11. A guy on our track team can high-jump 7 meters.
12. My friend ran 10,000 meters in less than an hour.
14. My car’s gas tank holds 12 meters of gasoline.
15. My daily food intake gives me about 10 million joules of energy.
16. Our utility company charges 10¢ per watt for the electricity we use.
17. The beach ball we played with had a density of 10 grams per cubic centimeter.
18. I know someone who went into a coma after drinking only three shots of hard liquor.

Basic Skills & Concepts


19. \(10^6 \times 10^5\)
20. \(10^4 \times 10^{-3}\)
21. \(\frac{10^6}{10^5}\)
22. \(\frac{10^7}{10^{10}}\)
23. \(10^{-2} \times 10^{-4}\)
24. \(10^{-7} \times 10^{-9}\)
25. \(10^6 + 10^5\)
26. \(10^7 - 10^2\)
27. \(10^{12} \times 10^8\)
28. \(10^6 \times 10^{-6}\)
29. \(\frac{10^{15}}{10^{12}}\)
30. \(\frac{10^{-7}}{10^4}\)
31. \(10^{-4} \times 10^{-6}\)
32. \(10^{-10} \times 10^{-14}\)
33. \(10^8 + 10^5\)
34. \(10^4 - 10^{-1}\)

USCS Units. Exercises 35–42 involve conversions within the U.S. customary system.

35. What is your height in inches?
36. What is your weight in ounces (avoirdupois)? in tons?
37. A gallon of water weighs about 128 ounces. How many pounds is that?
38. A boat is moving at 30 knots (nautical miles per hour). What is its speed in miles per hour?

39. You have a \( \frac{1}{2} \)-gallon milk jug. How many liquid pints can it hold? How many dry pints can it hold?

40. Most soda cans contain 12 fluid ounces. How many cubic inches do they contain?

41. If 150 million bushels of wheat are traded on one day, how many cubic inches are traded? How many cubic feet?

42. A small city produces 500,000 cubic feet of garbage per week. If all of this garbage were stacked neatly (in a nice vertical pile) on a 100-yard by 60-yard football field, how high would the pile be (in feet)?

Metric Prefixes. For each pair of units in Exercises 43–48, state how much larger or smaller the first unit is than the second.

43. millimeter, meter
44. milliliter, deciliter
45. gram, kilogram
46. kilometer, micrometer
47. square millimeter, square kilometer
48. cubic meter, cubic centimeter

USCS-Metric Conversions. Convert each measurement in Exercises 49–58 to the units specified.

49. 10 meters to feet
50. 105 centimeters to yards
51. 880 yards to kilometers
52. 150 pounds to kilograms
53. 20 gallons to liters
54. 100 kilometers per hour to miles per hour
55. 5 milliliters to cubic inches
56. 5.5 grams per cubic centimeter to pounds per cubic foot
57. 1200 square feet to square meters
58. 25 miles per hour to kilometers per hour

Celsius-Fahrenheit Conversions. In Exercises 59–60, convert Celsius temperatures into Fahrenheit or Fahrenheit temperatures into Celsius.

59. a. 45°F  b. 20°C  c. -15°C  d. -30°C  e. 70°F
60. a. -8°C  b. 15°F  c. 15°C  d. 75°F  e. 20°F

Celsius-Kelvin Conversions. In Exercises 61–62, convert Celsius temperatures into Kelvin or Kelvin temperatures into Celsius.

61. a. 50 K  b. 240 K  c. 10°C
62. a. -40°C  b. 400 K  c. 125°C

63. Basketball Power. You burn 800 Calories while playing in a basketball game for an hour. What is your average power during the game, in watts? Is it enough to keep a 100-watt light bulb shining? Explain.

64. Aerobics Power. You burn 500 Calories while doing aerobics for 45 minutes. What is your average power during the class, in watts? Is it enough to keep a 100-watt light bulb shining? Explain.

Electric Bills. For the electric bills described in Exercises 65–66, do the following:

65. a. Determine your total electrical energy use, in joules.
   b. Determine your average power use, in watts.
   c. Assuming the power company generated the energy burning oil, calculate the amount of oil needed to provide the energy shown on your bill. Give your answer in both liters and gallons. (Hint: Burning 1 liter of oil releases 12 million joules of energy.)

66. a. Your electric bill states that you used 1250 kilowatt-hours of energy in June.
   b. Your electric bill states that you used 970 kilowatt-hours of energy in September.

Densities. In Exercises 67–72, give the densities in appropriate units (population data from 2000 census).

67. A 40-gram pebble has a volume of 10 cubic centimeters. What is its density? Will it sink or float in water?

68. A jug has a total volume of 8 liters (which is 8000 cubic centimeters) and a mass of 6 kilograms. What is its density? Will it sink or float in water?

69. The land area of the United States is about 3.5 million square miles, and the population is about 260 million people. What is the average population density?

70. Find the population and the area of your home town. Calculate its population density.

71. New Jersey and Wyoming have areas of 7419 and 970,000 square miles, respectively, and population 8.4 million and 490,000, respectively. Calculate and compare their population densities.
72. A new computer hard drive holds 250 gigabytes of information on a surface area of 40 square centimeters. Calculate the information density on the disk.

73. Blood Alcohol Content: Wine. A typical glass of wine contains about 20 grams of alcohol. Consider a 110-pound woman, with approximately 4 liters (4000 milliliters) of blood, who drinks two glasses of wine.

a. If all the alcohol were immediately absorbed into her bloodstream, what would her blood alcohol content be? Explain why it is fortunate that, in reality, the alcohol is not absorbed immediately.

b. Again assume that all the alcohol is absorbed immediately, but now assume that her body eliminates the alcohol (through metabolism) at a rate of 10 grams per hour. What is her blood alcohol content 3 hours after drinking the wine? Is it safe for her to drive at this time? Explain.

74. Blood Alcohol Content: Hard Liquor. Eight ounces of a hard liquor (such as whiskey) typically contain about 70 grams of alcohol. Consider a 200-pound man, with approximately 6 liters (6000 milliliters) of blood, who quickly drinks 8 ounces of hard liquor.

a. If all the alcohol were immediately absorbed into his bloodstream, what would his blood alcohol content be? Explain why it is fortunate that, in reality, the alcohol is not absorbed immediately.

b. Again assume that all the alcohol is absorbed immediately, but now assume that his body eliminates the alcohol (through metabolism) at a rate of 15 grams per hour. What is his blood alcohol content 4 hours after drinking the liquor? Is it safe for him to drive at this time? Explain.

Further Applications

75. The Metric Mile. In track and field, the 1500-meter race is sometimes called the metric mile.

a. Compare the metric mile to a USCS mile. How much longer or shorter is it in terms of percentage?

b. Look up the current men's and women's world records for the mile. If you assume that the runners maintain the same pace for the metric mile, what should their times be for the metric mile? Compare your predicted records at this pace to the actual current world records for the metric mile.

76. Metric Tools. Many tools come in both USCS and metric standards. In a standard socket set, the smallest USCS subdivision is \( \frac{1}{16} \) inch; the smallest metric subdivision is 0.5 millimeter. Are the tools interchangeable? Explain.

77. Tallest Mountain? Mauna Kea, the highest mountain on the island of Hawaii, rises 13,796 feet above sea level. It extends an additional 18,200 feet from sea level to its base on the ocean floor. How tall is Mauna Kea from its base to its peak, in feet, miles, and kilometers? Compare its total extent to the height of Mt. Everest above sea level (29,023 ft). Would it be fair to call Mauna Kea the highest mountain in the world? Why or why not?

78. The Cullinan Diamond and the Star of Africa. The largest single rough diamond ever found, the Cullinan diamond, weighed 3106 carats. It was used to cut the world's largest diamond gem, the Star of Africa (530.2 carats), which is part of the British crown jewels collection. How much did the Cullinan diamond weigh in milligrams? In (avoirdupois) pounds? How much does the Star of Africa weigh in milligrams? In (avoirdupois) pounds?

Gems and Gold. Exercises 79–82 use karats and carats (see the Practical Matters box).

79. You find a nugget that is 25% gold. What is its purity in karats?

80. You purchase a 14-karat gold chain that weighs 15 grams. How much gold have you purchased (in grams)? Bonus: At the current price of gold, how much is the gold in the chain worth?

81. How much does a 2.5-carat diamond weigh in grams?

82. Is it possible to have jewelry made of 30-karat gold? Why or why not?

83. Refrigerator Cost. Your refrigerator uses an average power of 350 watts, and your utility company charges 8¢ per kilowatt-hour of energy. How much does it cost to run your refrigerator for a year? Explain.
84. **Hair Dryer Cost.** You have an 1800-watt hair dryer, which you use for an average of 10 minutes per day. Your utility company charges 9c per kilowatt-hour of energy. How much does it cost to run the hair dryer each day? Each year?

85. **Compact Fluorescent Light Bulbs.** You replace a 100-watt standard light bulb with a 25-watt compact fluorescent bulb that supplies the same light. Over a 10,000-hour life (typical for a compact fluorescent bulb), how much energy, in kilowatt-hours, do you save? If electricity costs 8c per kilowatt-hour, how much money do you save?

86. **Human Wattage.** Suppose you require 2500 food Calories per day (which is about average).
   a. What is your average power, in watts? Compare your answer to the wattage of some familiar appliance.
   b. How much energy, in joules, do you require from food in a year? Counting all forms of energy (such as gasoline, electricity, and energy for heating), the average U.S. citizen consumes about 400 billion joules of energy each year. Compare this value to the energy needed from food alone.

87. **Coal Power Plant.** A new coal-burning power plant can generate 1 gigawatt (billion watts) of power. Burning 1 kilogram of coal yields about 450 kilowatt-hours of energy. How much energy, in kilowatt-hours, can the plant generate each month? How much coal, in kilograms, is needed by this power plant each month? If a typical home uses 1000 kilowatt-hours per month, how many homes can this power plant supply with energy?

88. **Nuclear Power Plant.** Operating at full capacity, the Fort St. Vrain Nuclear Power Station in Colorado can generate 330 megawatts of power. Nuclear fission of 1 kilogram of uranium (in the form of uranium-235) releases 16 million kilowatt-hours of energy. How much energy, in kilowatt-hours, can the plant generate each month? How much uranium, in kilograms, is needed by this power plant each month? If a typical home uses 1000 kilowatt-hours per month, how many homes can this power plant supply with energy?

89. **Solar Energy.** Use the following facts in Exercises 89–90: Solar (photovoltaic) cells convert sunlight directly into electricity. If solar cells were 100% efficient, they would generate about 1000 watts of power per square meter of surface area when exposed to direct sunlight. With lower efficiency, they generate proportionally less power. For example, 10% efficient cells generate 100 watts of power in direct sunlight.

   a. Suppose a 1-square-meter panel of solar cells has an efficiency of 20% and receives the equivalent of 6 hours of direct sunlight per day. How much energy, in joules, can it produce each day? What average power, in watts, does the panel produce?

   b. Suppose you want to supply 1 kilowatt of power to a house (the average household power requirement) by putting solar panels on its roof. For the solar cells described in Exercise 89, how many square meters of solar panels would you need? Assume that you can make use of the average power of the solar cells (by, for example, storing the energy in batteries until it is needed).

90. **Wind Power: One Turbine.** Modern wind energy "farms" use large wind turbines to generate electricity from the wind. At a typical installation, a single modern turbine can produce an average power of about 200 kilowatts. (This average takes wind variations into account.) How much energy, in kilowatt-hours, can such a turbine generate in a year? Given that the average household uses about 10,000 kilowatt-hours of energy each year, how many households can be powered by a single wind turbine?

91. **California Wind Power.** California currently has wind farms capable of generating a total of about 2000 megawatts (2 gigawatts) of power.

   a. How much energy, in kilowatt-hours, can these wind farms generate each year? Given that the average household uses about 10,000 kilowatt-hours of energy each year, how many households can be powered by these wind farms?

   b. One of the great advantages of wind power is that it does not produce the carbon dioxide emissions that contribute to global warming. On average, energy produced from fossil fuels generates about 1.5 pounds of carbon dioxide for every kilowatt-hour of energy. Suppose California did not have its wind farms and the energy were instead produced from fossil fuels. How much more carbon dioxide would be entering the atmosphere each year?
Currency Conversions. Find today’s currency conversion rates, and use them to answer Exercises 93–96. Be sure to state clearly the currency conversion you use.

93. You see oranges in an Italian market priced at 0.60 euro per kilogram. What is the price in dollars per pound?
94. Gasoline at a German gas station costs 9.50 euros per liter. What is the price in dollars per gallon?
95. A French car manufacturer claims that its newest economy model has gas “mileage” of 15 kilometers per liter. What is the gas mileage in miles per gallon?
96. A supermarket in Mexico sells milk for 6.5 pesos per liter. What is the price in dollars per quart?
97. A fine ale in an English pub sells for 3.5 pounds (currency) per liter. What is the price in dollars per ounce?
98. A piece of land in the Belgian countryside with an area of 0.1 square kilometer is priced at 6000 euros. What is the price in dollars per acre?
99. Project: Personal Energy Audit. Do a thorough electrical energy audit of your home, apartment, or dormitory. That is, determine the energy used in a typical month by each of your electrical appliances, and your total energy usage per month for electricity. Based on the cost of electricity in your area, calculate the average monthly cost for electricity. After studying your findings, propose at least three energy conservation strategies that you would consider implementing to save money and energy. Discuss the pros and cons of each strategy.

Find the EPA standards for each pollutant, and find some of the hazards associated with exposure to each pollutant. Track how the levels of pollution in this city have changed over the past 20 years. Based on your findings, do you think it is likely that pollution in this city will get better or worse over the next decade? Summarize your findings and your conclusions in a one- or two-page report.

103. Alcohol Poisoning. Research some aspect of the dangers of alcohol, such as drunk driving or alcohol poisoning (in which a person dies from drinking too much too fast). Find statistics related to this issue, especially data that relate the blood alcohol content to dangers. Summarize your findings in a short report about how society might combat the danger.

104. Wind Power. Learn more about both the currently installed wind power and the future potential for wind power in your state. Overall, do you expect wind power to be important in the future in your state? Why or why not?

In the News

105. Everyday Metric. Describe three ways that you use metric units in your everyday life.
106. Should the United States Go Metric? Discuss the pros and cons of having the United States switch fully to the metric system. Do you think it will ever happen?
107. Energy. Look for a news article concerning energy or power. What units are used to describe energy or power? Summarize the article and explain the meaning of the units.
108. Density and Concentration. Look for a news article that uses units of density or concentration in any context. Summarize the article and explain the meaning of the units.
109. Utility Bill. Analyze a utility bill. Explain all the units shown, and determine the relative costs of different energy uses. What changes would you recommend if the recipient of the bill wanted to lower energy costs?
110. Pollution Problems. What (if any) pollutants are of concern in the area where you live? What units are used to describe their concentrations? At what concentrations are the pollutants considered dangerous? Suggest ways that these pollution problems might be addressed.

Web Projects

Find useful links for Web Projects on the text Web site: www.aw.com/bennett-briggs

100. Metric History. Research some aspect of the history of the metric system, such as how it came to be adopted around the world or how various units have been scientifically defined. Write a short report on your findings.

101. Energy Issues. The question of how we will continue to meet our energy needs is one of the most important issues of our time. Research one aspect of this issue using information available at the Web site for the U.S. Energy Information Administration. Write a short report on your findings.

102. Pollution Progress. Investigate the average concentrations of various pollutants in a major city of your choice.
Time out to think
China occupies roughly the same amount of land as the United States, but has more
than four times as many people. Given these circumstances, do you think that the one-
child policy is a good idea? Would a one-son policy be better? Defend your opinions.

HINT 8: DO NOT SPIN YOUR WHEELS
Finally, everyone has had the experience of getting "bogged down" with a problem.
When your wheels are spinning, let up on the gas! Often the best strategy in problem
solving is to put a problem aside for a few hours or days. You may be amazed at what
you see (and what you overlooked) when you return to it.

EXERCISES 2C

Review Questions
1. Describe the four basic steps of problem solving.
2. Summarize the strategic hints for problem solving given
   in this unit, with an example of the meaning of each one.

Does It Make Sense?
Decide whether each of the following statements makes
sense (or is clearly true) or does not make sense (or is clearly
false). Explain your reasoning.
3. If you follow my simple problem-solving recipe, you will
   be able to solve any mathematical problem easily and
   with a minimal investment of time.
4. Whether it's a problem in mathematics or something else,
   it's best to start by taking time to make sure you under-
   stand the nature of the problem.
5. If your answer doesn't make sense, then you must not
   have solved the problem correctly.
6. Kids are often taught to draw pictures for simple math
   problems, but grownups should be able to solve problems
   without any pictures.

Basic Skills & Concepts
7. Box Office. You work at a box office selling tickets that
   cost $15 for children and $30 for adults. You sold a total of
   $150 worth of tickets, but did not keep track of child and
   adult tickets separately. Can you determine how many
   child and how many adult tickets you sold? Find all the
   possible solutions to the problem.

8. Party Punch. You're making up some party punch using
   cranberry juice and ginger ale. The cranberry juice comes
   in 4-quart bottles, and the ginger ale comes in 2-quart
   bottles. You mix a huge 48-quart (12-gallon) batch of
   punch, but forget how many bottles of cranberry juice and
   ginger ale you poured in. Can you find the exact recipe for
   the punch? Find all possible solutions to the problem.

More on Jack and Jill. Refer to Example 2 for Exercises 9–10.
9. Suppose that Jill's time in the first race was 10 seconds.
   Using strategy 1, determine by how much she would
   win the second race.
10. Hack and Quill race 200 meters and Hack wins by 10 meters.
    They race a second time, with Hack starting 10 meters
    behind the starting line. Who wins the second race?
    Explain.

11. Cars and Canary. Refer to Example 3. Two cars, 150 kilo-
    meters apart, begin driving toward each other on a long,
    straight highway. One car travels 80 kilometers per hour,
    the other 100 kilometers per hour. At the same time, a
    canary, starting on one car, flies back and forth between
    the two cars as they approach each other. If the canary flies
    120 kilometers per hour and spends no time to turn around
    at each car, how far has it flown when the cars collide?
12. Cars and Canary. Repeat Exercise 11, but assume the
    canary flies at only 60 kilometers per hour.

13. Mixing Marbles. Refer to Example 4. Consider the case
    in which each pile initially has fifteen marbles. Suppose
    that on the first transfer three black marbles are moved to
    the white pile. On the second transfer, any three marbles
    are taken from the white pile and put into the black pile.
    Demonstrate, with diagrams and words, that you will
    always end up with as many white marbles in the black
    pile as black marbles in the white pile.
14. Mixing Marbles. Repeat Exercise 13, but for the case of piles of twenty marbles and transfers of four marbles at a time.

15. Wedding Decorations. You are responsible for wrapping ribbon around the cylindrical columns in a large banquet room that will be the site of a wedding reception. The room has 20 columns that are each 12 feet high and have a circumference of 3 feet. You plan to make ten wraps of ribbon around each column. How much ribbon will you need?

16. Coiled Wire. Eight turns of a wire are wrapped around a pipe with a length of 20 centimeters and a circumference of 6 centimeters. What is the length of the wire?

Bowed Rail. Exercises 17–18 refer to Example 6 in the text.

17. In a few sentences, briefly explain why the triangle makes a good approximation to the actual shape of the bowed rail and why a circle with radius $\frac{1}{2}$ mile would not have made a good approximation.

18. Suppose the rail is 1 kilometer long, and it expands on a hot day by 10 centimeters in length. Approximately how high would the center of the rail rise above the ground?

19. China's One-Child Policy. To convince yourself that a one-child policy would lead to an average of two children per family, with equal numbers of boys and girls, do the following. Suppose that 10,000 families are having children according to the one-child policy. Describe the general makeup of all of the families (that is, start with the fact that 5000 families have a boy as their first and therefore only child, and continue on). Use this process to show that the average number of children is two and that boys and girls are equal in number.

20. Demographic Imbalance. Suppose that, as the current generation matures, China's population of young adults has more men than women by a ratio of 118 to 100. With 400 million young adults in China, how many men will be unable to find a spouse?

Further Applications

21. Gardening Supplies. Topsoil comes in 3-cubic-foot bags, and fertilizer comes in 1-cubic-foot bags. Assuming you use entire bags (so that you do not waste any soil or fertilizer), how many different ways can you make a 25-cubic-foot mixture of soil and fertilizer? Find all the possible solutions to the problem.

22. Traffic Counter. A traffic counter is a device designed to count the number of vehicles passing along a street. It usually is a thin black tube stretched across a street or high-way, connected to a "brain box" at the side of the road. The device registers one "count" each time a set of wheels (that is, wheels on a single axle) rolls over the tube. A normal automobile registers two counts: one for the front wheels and one for the rear wheels. A truck with three axles (front wheels plus a double set of rear wheels) registers three counts. A large semitrailer truck might have four or five axles and hence register four or five counts. Suppose that, during a 1-hour period, a traffic counter registered 35 counts on a residential street on which only two-axle vehicles (cars) and three-axle vehicles (light trucks) are allowed. How many cars and light trucks passed over the traffic counter? Find all the possible solutions to the problem.

23. Stereo Wire 1. A stereo system is being installed in a room with a rectangular floor measuring 12 feet by 10 feet and an 8-foot ceiling. The stereo amplifier is near one corner of the room, 1 foot from the floor and 1 foot from a side wall. A speaker is near the opposite corner of the room, 1 foot from the ceiling and 1 foot from the opposite wall. You must run a wire from the amplifier to the speaker, and the wire must run along the floor or walls (not through the air). What is the minimum length of wire you can use for the connection? (Hint: Turn the problem into an equivalent similar problem by imagining cutting the room along its vertical corners and unfolding it so that it is flat. You will be able to apply the Pythagorean theorem.)

24. Stereo Wire 2. You are connecting the same amplifier and speaker described in Exercise 23. This time, however, the wire must run along the edges of the room for as much of the distance as possible. (That is, you cannot run it diagonally across the floor.) What is the minimum amount of wire needed for the connection?

25. Bridge Height. A curved bridge rises over a river. The two endpoints of the bridge are 100 meters apart (horizontally). You walk across the bridge with a device to measure its length, and you discover that the walking distance is 102 meters. Approximately how high does the bridge rise over the river? (Hint: See Example 6.)

26. Bridge Length. A curved bridge rises over a small creek. The two endpoints of the bridge are 25 meters apart (horizontally). The bridge rises to a height of 3 meters (above the horizontal). Approximately what is the walking distance along the bridge? (Hint: See Example 6, and note that this time you are given the height and must determine the hypotenuse.)

Puzzle Problems. Exercises 27–41 are puzzle problems that require careful thinking and can help your problem-solving skills. Hint: Look for "ahah" solutions.
27. It takes you 30 seconds to walk from the first (ground) floor of a building to the third floor. How long will it take to walk from the first floor to the sixth floor (at the same pace, assuming that all floors have the same height)?

28. Reuben says, "Two days ago I was 20 years old. Later next year I will be 23 years old." How is this possible?

29. There are three kinds of apples all mixed up in a basket. How many apples must you draw (without looking) from the basket to be sure of getting at least two of one kind?

30. "Brothers and sisters I have none, but that man's father is my father's son." Who is "that man"?

31. "I am the brother of the blind fiddler, but brothers I have none." How can this be?

32. A woman bought a horse for $500 and then sold it for $600. She bought it back for $700 and then sold it again for $800. How much did she gain or lose on these transactions?

33. Three boxes of fruit are labeled Apples, Oranges, and Apples and Oranges. Each label is wrong. By selecting just one fruit from just one box, how can you determine the correct labeling of the boxes?

34. Each of ten large barrels is filled with golf balls that all look alike. The balls in nine of the barrels weigh 1 ounce and the balls in one of the barrels weigh 2 ounces. With only one weighing on a scale, how can you determine which barrel contains the heavy golf balls?

35. A woman is traveling with a wolf, a goose, and a mouse. She must cross a river in a boat that will hold only herself and one other animal. If left to themselves, the wolf will eat the goose and the goose will eat the mouse. How many crossings are required to get all four creatures across the river alive?

36. How do you measure 9 minutes with a 7-minute and a 4-minute hourglass? Assume the hourglasses can only measure 7-minute and 4-minute intervals and cannot be used to measure other time intervals (for example, you can’t tell when 2 minutes have gone by).

37. A 150-foot rope is suspended at its two ends from the tops of two 100-foot flagpoles. The lowest point of the rope is 25 feet from the ground. What is the distance between the flagpoles?

38. You are considering buying 12 gold coins that look alike, but you have been told that one of them is a heavy counterfeit. How can you find the heavy coin in three weighings on a balance scale?

39. Suppose you have 40 blue socks and 40 brown socks in a drawer. How many socks must you take from the drawer (without looking) to be sure of getting a pair of the same color?

40. Alma, Bess, Cleo, and Dina visited Elf on Saturday. Alma visited at 8:00, Bess visited at 9:00, Cleo visited at 10:00, and Dina visited at 11:00. Some clues: (1) The times may be either A.M. or P.M. (2) At least one woman visited Elf between Alma and Bess. (3) Alma did not visit Elf before both Cleo and Dina. (4) Cleo did not visit Elf between Bess and Dina. Who visited Elf last?

41. Three prisoners know that the jailer has three white hats and two red hats. The jailer gives one hat to each prisoner and says, "If you can deduce the color of your own hat, you will be freed." Each prisoner can see the hats of the other two prisoners, but not his own. The first prisoner says, "I cannot tell the color of my hat." The second prisoner says, "I cannot tell the color of my hat." The third prisoner is blind, but he is freed. What color hat does he have and how did he know?

Projects: Real Problems in the Real World. Exercises 42–50 describe complex problems that do not have single, straightforward solutions. For each, describe how you would apply the four-step problem-solving process described in the text (without actually carrying out the process to obtain a solution). You may choose either to enumerate the four steps or to describe your process in essay form. Either way, be sure that you list as many relevant factors as possible and discuss sources of uncertainty associated with each factor. Also, describe how you would work with these factors to find a solution. Conclude by describing your overall impression of whether the problem can be solved and whether any solution would likely generate controversy.

42. You are asked to calculate the cost of installing enough bike racks on campus to solve a bicycle parking problem.

43. You want to know how many new faculty members would be needed, and the total cost to the university, to make sure that all classes have twenty or fewer students (to replace large lecture classes with smaller classes).

44. You want to know whether having a top-quality football program means more money for academics at a university.

45. You want to figure out how much taxes would have to be increased to provide public school teachers with twice their present salaries.

46. You decide that, in the interest of the environment, you will convert your home heating and hot water system to
solar power. How much will this conversion cost or save over the next ten years?

47. Suppose that China and India decide to use their extensive coal reserves to supply energy to their populations at the same per capita level as in the United States. How much carbon dioxide would be added to the atmosphere?

48. Are automobile insurance companies gouging drivers? Suppose that you want to figure out whether they are justified in raising insurance rates as rapidly as they have during the past few years.

49. A large city is facing a severe shortage of water. How should water be rationed?

50. Suppose that a city added new bus routes and handed out free bus passes. How many people would give up driving in favor of the bus? How much money, overall, would this cost or save the city?

51. Project: Textbook Analysis. Although research shows that most adults today have difficulty with “story problems,” we might hope that the next generation will have less difficulty. Find a current textbook in mathematics that is used at the upper elementary school level (grades 4–6). Read through the “story problems” in the textbook. Write a critical analysis of the problems and conclude with an opinion as to whether the problems make mathematics meaningful.

Web Projects

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52. China’s Population. Find current statistics regarding China’s population, such as its total population and average number of children per family, and projections of its future population. Based on what you find, does it appear that China is slowing its rate of population growth? Explain.

53. Polya and Problem Solving. Many Web sites discuss George Polya’s work and other problem-solving strategies. Explore a few such sites. Write a one- or two-paragraph summary of something you learn about problem solving that you think would be useful to many people.

In the News

54. Multiple Solutions. Find an example of a real problem for which, because of insufficient data, we cannot distinguish between two or more possible solutions. The problem might come from a news report or from your own experiences. What additional data would be useful?

55. Multiple Strategies. Find an example of a real problem that could potentially be solved by two or more competing strategies. The problem might come from a news report or from your own experiences. Describe each strategy. Which one do you think is better? Why?

56. Novel Solution. Find a news report concerning a problem in business or science that was solved by a surprising method. Describe the method and why it was useful.

57. China’s Population Policy. Look for a recent news report concerning consequences of China’s population policies. Does the news report affect your overall opinion of China’s policy? Explain.